# WEAK INTERACTION STRUCTURE STUDIES THROUGH NEUTRINO REACTIONS AT HIGH ENERGIES

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*By*

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to the

**DEPARTMENT OF PHYSICS**

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**MARCH, 1973**

*to my mother*

# CERTIFICATE

Certified that the work presented in this thesis entitled " Weak Interaction Structure Studies through Neutrino Reactions at High Energies" by S. Krishna has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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# SYNOPSIS

Thesis entitled 'Weak Interaction Structure Studies through Neutrino Reactions at High Energies', submitted by SRINIVAS KRISHNA in partial fulfilment of the requirements of the Ph.D. degree to the Department of Physics, Indian Institute of Technology, Kanpur.
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The structure of the weak interaction Lagrangian has been a subject of much experimental and theoretical investigation. The processes which were studied till now were all at low energies and small momentum transfers and were well described by the local current-current weak Lagrangian. In the attempt to build a more complete theory of weak interactions, a first step would be to look for deviations from the implications of the local current-current Lagrangian. Important results in this regard are the lepton pair locality theorems of Pais and Treiman which give cross sections for neutrino processes as explicit functions of two lepton system variables. These theorems are valid to the lowest order in the weak coupling constant $G$ in the local current theory as well as in the vector boson mediated theories.

However, renormalizable models of weak interactions have been constructed in which the lowest order weak process

is the result of a fourth order interaction involving exchange of two scalar bosons. In these models the lepton pair is not local even in the lowest order weak coupling. Substantial deviations from the Energy Theorem of Pais and Treiman are thus shown to occur for quasielastic neutrino scattering. For the angle dependence theorem deviations seem to be small.

Even when the local current-current Lagrangian can describe the weak couplings, deviations from lepton pair locality occur due to electromagnetic interactions. For processes involving emission of a single photon, however, the expressions are modified in a definite fashion and the cross sections can, therefore, still be given as explicit functions of two lepton system variables.

The theorems giving lepton energy and angle dependence for radiative neutrino reactions are also violated in the scalar boson mediated theories. In this case, since photon can couple to the charged lines in the box diagram typical of scalar theories, deviations from lepton pair locality theorems can be calculated without detailed assumptions concerning the hadronic weak currents, retaining only certain asymptotic expressions for the time ordered products of these currents. Specific values have been calculated in this fashion for the reaction with the nucleon only as the final state hadron. Substantial violations of the Energy theorem occur but deviations from the angle theorem are again small.

# CHAPTER - I

## INTRODUCTION

Our knowledge of weak interactions seems to be at present at the threshold of a new phase of activity, both in experiment and theory. The history of this field dates back to the turn of the last century when beta decay of nuclei was discovered[1]. The existence of the neutrino was postulated by W. Pauli in 1933. In 1934 Fermi proposed what is even now the basis of much of weak interaction theory- the four fermion interaction. The next phase of activity was in the latter half of the fifties when parity violation in weak interactions was discovered and Vector-Axial vector nature of the hadronic weak currents established. In the current phase the main interest is in going beyond the current-current theory towards a more complete theory of weak interactions. The current resurgence of interest owes a good deal to the recent commencement of operations by the 500 GeV accelerator at Illinois and also of a new generation of big bubble chambers. With the expectation of the possibilities implied by the proposed neutrino scattering experiments at high energies, the structure of the weak interaction Lagrangian has become a topic of renewed interest.

The observed weak processes, which are all at low energies and small momentum transfers, can be understood in terms of a local current-current form for the weak interaction Lagrangian.

$$L_{eff} = \frac{G}{\sqrt{2}} (J_\lambda + \ell_\lambda) \; (J_\lambda + \ell_\lambda)^\dagger \tag{1}$$

G is the weak interaction coupling constant.
$G = 1.026 \times 10^{-5} \; m_p^{-2}$,. $\ell_\lambda$ is the lepton weak current formed from the muon, electron and the corresponding neutrino fields.

$$\ell_\lambda = \bar{\Psi}_{\nu_\mu} \gamma_\lambda \; (1 - \gamma_5) \; \Psi_\mu + \bar{\Psi}_{\nu_e} \; \gamma_\lambda (1 - \gamma_5) \; \Psi_e$$

$J_\lambda$ is the hadronic weak current containing a vector and an axial vector current in a V-A combination and containing a strangeness preserving and a strangeness changing part with a relative weight determined by the cabibbo angle $\theta_c$

$$J_\lambda = (V_\lambda - A_\lambda)^{\Delta S=0, \Delta I=1} \operatorname{Cos} \theta_c + (V_\lambda - A_\lambda)^{\Delta S=1, \Delta I = \frac{1}{2}} \operatorname{Sin} \theta_c$$

According to the remarkably successful current algebra postulates, the hadronic currents are assigned to an SU(3) x SU(3) algebra and obey the Gell-Mann commutation relations

$$[\, V_{oa}(x),\; V_{ob}(y) \,]_{x_o = y_o} = i \, \varepsilon_{abc} \, V_o^{\,c} \; (x) \; \delta(\vec{x} - \vec{y})$$

$$[\, A_{oa}(x),\; A_{ob}(y) \,]_{x_o = y_o} = i \, \varepsilon_{abc} \, V_o^{\,c} \; (x) \; \delta(\vec{x} - \vec{y})$$

$$[\, V_{oa}(x),\; A_{ob}(y) \,]_{x_o = y_o} = i \, \varepsilon_{abc} \, A_o^{\,c} \; (x) \; \delta(\vec{x} - \vec{y})$$

The main shortcoming of the current-current Lagrangian which precludes consideration of (1) as the actual field theoretic Lagrangian rather than a phenomenological lagrangian

is the non-renormalizability of the four fermion interaction. But a phenomenological lagrangian cannot be valid at all energies. The four fermion interaction, therefore, violates unitarity at sufficiently high energies as pointed out by Heisenberg soon after Fermi's proposal. This can be seen easily by considering the reaction

$$\nu_\ell + \ell^- \rightarrow \nu_\ell + \ell^-$$

For energies large enough, the lepton mass can be neglected. The differential cross-section then is

$$\frac{d\sigma}{d\Omega} \simeq \frac{G^2}{\pi^2} p_{cm}^2$$

where $p_{cm}$ is the centre of mass momentum. With the usual partial wave decomposition, this corresponds to a pure S wave scattering with an S wave partial amplitude

$$<\lambda_3\lambda_4|T^0|\lambda_1\lambda_2> = \frac{G}{\pi} 2\, p_{cm}^2$$

where $\lambda$'s are the helicities of the particles involved. The requirement of unitarity of the S-matrix implies the condition

$$|<\lambda_3\lambda_4|T^0|\lambda_1\lambda_2>| < 1$$

This limit is violated for sufficiently large values of $p_{cm}$. This will be at energies around 350 GeV, the so called unitarity limit.

The main effort in weak interaction theory, therefore, is directed towards obtaining a Lagrangian which would give $\mathcal{L}_{eff}$ in the approximation of small values for the kinematic variables involved. The simplest such model is one in which an additional vector field $W_\lambda$ is introduced.

$$\mathcal{L}_V = g \{\ell^\lambda W_\lambda + J^\lambda W_\lambda\}$$

For energies and momentum transfers small compared with the mass $M_W$ of the $W$ boson, this leads, in the second order, to the same matrix elements as $\mathcal{L}_{eff}$ for the familiar weak processes, if the coupling constant $g$ and the mass $M_W$ are related to the weak interaction coupling constant $G$ by

$$\frac{g^2}{M_W{}^2} = \frac{G}{\sqrt{2}} .$$

A simple vector boson mediated weak interaction theory is not renormalizable either. So there were various attempts to construct a renormalizable theory of weak interactions mediated by vector bosons. One such was the Gell-Mann, Goldberger, Kroll and Low[2] theory in which scalar bosons are introduced with a gradient coupling so as to cancel the divergences for the so-called non-diagonal processes. But the GGKL model is not renormalizable either, the divergences merely being transferred to the diagonal interactions. Another attempt is the strongly interacting intermediate boson theory of Marshak, Okubo and coworkers[3] in which the assumed strong

interactions of the intermediate boson provide a 'natural' cutoff at $\Lambda \approx M_W$.

The most promising and elegant formulation of a vector boson mediated weak interaction theory is in the framework of the Gauge theories. Such a theory of leptonic interactions was originally proposed by Weinberg and by Salam[4]. In such theories, a neat unification of the weak and electromagnetic interactions is also achieved. The current resurgence of interest in these theories follows t'hooft's work concerning renormalizability of the Gauge theories. At present considerable attention is focussed on proving the renormalizability of such theories and also in extending the theory to interactions of hadrons[5].

In the present work, our main concern is with another class of theories, the theory of weak interactions mediated by scalar bosons. It was originally pointed out by Kummer and Segre[6] that models of weak interactions can be constructed in which the current-current interaction is obtained in the fourth order of an interaction mediated by scalar bosons. A later version of such a theory is due to N. Christ[7]. These models described the leptonic weak interactions neatly, could describe the semileptonic reactions also, but encountered trouble in the description of non-leptonic reactions even in the simplest versions, the main stumbling block being the large

parity and strangeness violations. Patil and Vaishya[8] could ameliorate such troubles through introduction of different coupling constants for the coupling of the leptons and the hadrons to the scalar boson. The most satisfactory formulation of a scalar boson mediated weak interaction theory, however, is due to V. Gupta and S.H. Patil[9]. By assuming different coupling constants for the leptons and hadrons to the scalar boson, assigning octet transformation properties to the hadronic scalar and pseudoscalar currents and formulating a universality criterion to determine the various coupling constants, Gupta and Patil were able to formulate a model with the least possible number of new particles and do away entirely with exotic hadrons and other miscellaneous particles which plagued earlier theories. We have mainly used the Gupta-Patil model as an illustration in calculating results typical of scalar boson mediated theories. We give in the next paragraph the Lagrangian of this model. Further details regarding the scalar boson mediated theories are relegated to the appendix A.

The Lagrangian of the Gupta-Patil model is

$$L_W = g_+ W^+ \{(S_2^1 - P_2^1) \operatorname{Cos} \theta_c + (S_3^1 - P_3^1) \operatorname{Sin} \theta_c\}$$

$$+ g W^- \sum_{\ell=e,\mu} L_\ell^- (1-\gamma_5)\, \nu_\ell + g' \left(\frac{\pi^o}{\sqrt{2}} + \frac{\eta^o}{\sqrt{2}} + \frac{X^o}{\sqrt{3}}\right) \sum_{\ell=e,\mu} L_\ell^- (1-\gamma_5)\, \ell^-$$

$$+ \text{H.c} + g_\pi \sum_{\alpha,\beta=1}^{3} \pi_\beta^\alpha P_\alpha^\beta$$

Here $W^{\pm}$ are heavy scalar intermediate bosons, $L_e^-$ and $L_\mu^-$ are charged heavy leptons, $S_\beta^\alpha$ and $P_\beta^\alpha$ $(\alpha, \beta = 1,2,3)$ are scalar and pseudoscalar hadronic currents. The $P_\beta^\alpha$ are also the sources of the nine pseudoscalar mesons $\pi_\beta^\alpha$ containing the octet $\pi^o$, $X^o$, etc. The third term couples the hadrons $\pi^o$, $\eta^o$ and $X^o$, which act as the 'neutral intermediate bosons', directly to the leptons. The fourth term is the SU(3)-symmetric strong interaction of the pseudoscalar mesons with the hadrons. Further, $g_\pi = \sqrt{2}\, g_{NN\pi}$ is the pion-nucleon coupling constant.

$$g' = g, \; g_+ g_\pi = g^2 ,$$

and

$$\left( \frac{g^2}{4\pi} \right)^2 \frac{1}{2M^2} = \frac{G}{\sqrt{2}} .$$

The charged heavy leptons and the heavy scalar intermediate bosons are assumed, for the sake of simplicity, to have the same mass M.

Our emphasis in subsequent chapters is on the implications of the local current-current Lagrangian, deviations from which may give unambiguous clues towards a more complete theory. In Chapter II we describe the lepton pair locality theorems of Pais and Treiman which obtain cross-sections for neutrino processes as explicit functions of two lepton system variables, assuming a local current-current weak lagrangian for semileptonic reactions.

Whereas at energies of around 300 GeV the local current-current effective lagrangian will not be sufficient to describe the weak processes, at energies of around 40 - 50 GeV, presumably only lowest order terms in G may be retained. We point out in Chapter III that only in scalar boson mediated weak interaction theories, violations of lepton pair locality theorems occur to the first order in G. We give numerical estimates of such violations for the quasi-elastic reaction and discuss troubles encountered in calculating such violations for reactions with additional hadrons.

Even when only the lowest order weak processes are dominant, violations of lepton pair locality theorems will occur due to electromagnetic interactions. In Chapter IV, we show that for reactions in which a single photon is emitted, which are reactions to the lowest order in weak and electromagnetic interactions, the local current-current weak lagraigian still implies explicit dependence on lepton energy and angle variables. The expressions of Pais and Treiman are modified by addition of well-defined terms.

In Chapter V, violations of lepton pair locality theorems for radiative neutrino reactions occurring in scalar boson mediated weak interaction theories is discussed. We point out that in contrast to the violations of the Pais-Treiman theorems, violations of the theorems for radiative scattering can be

calculated independent of the details of the weak currents of hadrons. We give a formal estimate of such violations for radiative reactions with an arbitrary hadronic system in the final state and consider reactions with only a nucleon in the final state in greater detail.

In the final and concluding Chapter, we have some comments on some related topics. We point out the possibility of calculating violations of lepton pair locality theorems due to higher order interactions in vector boson mediated theories in the framework of the spontaneously broken gauge theories. We also discuss possible violations due to weak resonances and presence of heavy leptons.

# CHAPTER - II

## LEPTON PAIR LOCALITY THEOREMS FOR NEUTRINO PROCESSES

This chapter is based mainly on a paper by A. Pais and S.B. Treiman entitled, "The full content of lepton pair locality in neutrino processes"[10]. In probing the structure of weak interactions, lepton pair locality theorems are of considerable importance. Pais and Treiman have shown that explicit expressions for lepton energy and angle dependence of cross-sections for neutrino reactions can be derived as a consequence of lepton pair locality. Their results are expected to hold to the lowest order in weak interaction coupling constant in a local current theory and also in vector boson mediated models of weak interactions but are violated, as we show in the next chapter, in scalar boson mediated theories.

Earlier tests for locality involved either averages over the lepton polarization or fixed lepton helicity. Besides these tests were of the 'quasi two body' reaction involving averaging over all variables of the hadronic system except for its invariant mass. In the Pais and Treiman version an arbitrary number of particles in the final state may be involved. Besides, the theorems may be derived either for specific helicities or for averages over polarizations.

A neutrino or antineutrino scattering process with $n$ hadrons in the final state is considered.

$$\nu_l(\bar{\nu}_l) + T \rightarrow h_1 + \cdots + h_n + l(\bar{l}) \qquad (1)$$

$T$ here is the target which may or may not be a single nucleon. Let $q_1$, $p$ and $q_2$ be the four momenta of the initial lepton, target and the final lepton respectively and $p_1 \ldots p_n$ be the momenta of the final hadrons. The cross section is a function of 3n-1 variables. Pais and Treiman work in the rest frame of the final state hadronic complex. We choose to work here in the laboratory frame ( $\vec{p} = 0$ ). Of the 3n-1 independent variables three are chosen to be

(1) E, the laboratory energy of the incident neutrino

(2) $t = (q_2-q_1)^2$ square of the momentum transfer from the leptons

(3) $s = (p_1 + \cdots + p_n)^2$ the invariant mass of the final state hadronic complex.

We define $L = q_2-q_1$ and $M = q_2 + q_1$. We choose the $z$ direction along $\vec{L}$, the $y$ axis along $\vec{L} \times \vec{Q}$ where $\vec{Q}$ is some vector of the final state hadronic complex and the $x$ axis along $(\vec{L} \times \vec{Q}) \times \vec{L}$. Further let $\phi$ be the angle between the $x$ axis and the projection of $\vec{N}$ on the $xy$ plane.

E, t, s and $\phi$ are chosen to be four of the 3n-1 independent variables. The rest of the 3n-5 independent

variables are constructed out of the $n$ momenta $p_1..p_n$ of the final state hadronic complex. The $n$ hadronic vectors $p_1...p_n$ have $3n$ independent components. Energy-momentum conservation restricts their sum $P = p_1+p_2+..+p_n$ to be equal to $q_1+p-q_2$. Further one vector $\vec{?}$ of the hadronic complex is constrained to the $xz$ plane. We have, therefore, in all five constraints. There are, therefore, $3n-5$ independent variables internal to the final state hadronic complex.

The matrix element for the neutrino scattering process with the phenomenological Lagrangian is given by

$$T_{fi} = \frac{G}{\sqrt{2}} \bar{u}(q_2) \gamma^\lambda (1-\gamma_5) u(q_1) < \alpha | J_\lambda | T >$$

where $J_\lambda$ is the hadronic V-A current and $\alpha$ is the final state hadronic complex.

The differential cross section averaged over all the hadron and lepton spins is

$$d\sigma = T_{\mu\nu} \tau^{\mu\nu} d\Omega$$

where $d\Omega$ is the phase space element (details in appendix B) and

$$T_{\mu\nu} = \sum_{\text{spins}} <\alpha|J_\mu|T> <\alpha|J_\nu|T>^\dagger$$

$$\tau_{\mu\nu} = \frac{1}{m_\ell E} \{N_\mu N_\nu - L_\mu L_\nu + (t-m_\ell^2) g_{\mu\nu} + i\varepsilon_{\alpha\beta\mu\nu} L^\alpha N^\beta\}$$

The crucial observation is that the hadronic factor cannot depend on $q_1$ or $q_2$ except insofar as the hadronic momenta are related to the leptonic momenta through the overall conservation law. Thus $T_{\mu\nu}$ can depend on L but not on N. The dependence of the cross section on N is, therefore, only through $\tau_{\mu\nu}$. Now the four vector N has two constraints $N^2 = -t + 2m_l^2$ and $N \cdot L = m_l^2$. Therefore, two lepton variables can be chosen such that the cross section dependence on them comes through the N terms in $\tau_{\mu\nu}$ only. We choose these two to be E and $\phi$. The components of N and L are (appendix B)

$$L_x = L_y = 0 \qquad L_z = \frac{1}{m_N}(Y^2 - tm_N^2)^{1/2} \qquad L_o = \frac{1}{m_N} Y \quad \text{where}$$

$$Y = \tfrac{1}{2}(m_N^2 + t - s)$$

$$N_x = n \operatorname{Cos} \phi \quad N_y = n \operatorname{Sin} \phi \qquad N_z = \frac{1}{m_N(Y^2 - tm_N^2)^{1/2}} \times \left[ Y^2 + 2EY\, m_N - m_l^2\, m_N^2 \right]$$

and $N_o = 2E + L_o$

$$\text{where} \quad n^2 = t - 2m_l^2 - \frac{m_N^2}{(Y^2 - tm_l^2)} \left[ tN_o^2 + m_l^4 - 2m_l^2 N_o L_o \right]$$

The dependence of the cross sections on E and $\phi$ is obtained by substituting for the components of N and L and multiplying by the appropriate phase space factor. We state the results

below.

The Energy Theorem: The dependence of the differential cross section on the energy of the incident neutrino is given, after integrating over the rest the phase space including the $\phi$ domain by

$$\frac{d^2\sigma}{ds\,dt} = \frac{G^2}{32\pi m_N^2 E^2} \left[ A E^2 + BE + C \right]$$

where A, B and C are functions of s and t only. The energy theorem was first derived by Lee and Yang[11].

The Angle Theorem: The cross section integrated over all the final state variables except $\phi$ is given by

$$\frac{d\sigma}{d\phi} = a \cos 2\phi + b \sin 2\phi + c \cos \phi + d \sin \phi + e$$

The above results are with summation over all hadron and lepton spins. Similar results can be derived for reactions with definite helicity.

This explicit two variable dependence is, according to Pais and Treiman, the maximum content of lepton pair locality since regardless of the complexity of the final state, there will never be more than two variables whose dependence is explicit.

The results, as is obvious from the derivation, are valid in the phenomenological current-current theory and also in any model in which weak interaction to order $G$ occurs through a single particle exchange.

# CHAPTER - III

## TESTS FOR SCALAR BOSON MEDIATED WEAK INTERACTION THEORIES IN HIGH ENERGY NEUTRINO SCATTERING PROCESSES

Search for a boson mediating weak interactions started soon after Yukawa's hypothesis of an intermediate boson for weak interactions in connection with his meson theory of nuclear forces. Successive experiments have not discovered any such boson, the current lower limit for the mass of such a boson being around 2 GeV[12]. A straightforward test for the scalar or vector boson mediated theory would involve detection of the intermediate boson and tests to determine its spin and other properties. Experimental searches are currently underway at the newer accelerators for the intermediate boson. But considering the large mass and decay rates such a particle is expected to have, these experiments may not succeed in unraveling all the information about these particles. Besides a conclusive test for any of the theories would involve more than detection and tests on the boson. Scalar bosons and heavy leptons, for example, are necessary components of more than one kind of theory. Indirect tests of a definitive nature are, therefore, of considerable interest. Such tests may also help in establishing the validity or otherwise of the theories at comparatively lower energies. This is a very important consideration, since in most models, the mass of the boson can be pushed without too much effort to values above the experimentally accessible range.

One such test is the observation by Mani and Vaishya[13] that Adler's theorem for the conservation of the vector current is violated in scalar boson mediated theories.

Adler[14] has shown that for inelastic $\Delta S = 0$ neutrino reaction

$$\nu_\ell + \alpha \to \beta + \ell$$

where $\alpha$ and $\beta$ are hadrons, the conservation of the hadronic vector current implies absence of parity violating effects in the parallel configuration in which the four momentum of the outgoing lepton is parallel to that of the incident neutrino. In the Patil-Vaishya[8] version of the scalar theory, for the reaction

$$\nu_\ell + n \to p + \ell^- + \eta^o$$

with a scalar boson of mass 6 GeV parity violating effects of the order of 10 per cent were shown to occur for neutrino energies around 10 GeV.

In subsequent sections of this chapter we consider a test for the scalar boson mediated weak interaction theories which involve observation of deviations from the explicit two variable dependence of cross section obtained as a consequence of lepton pair locality by Pais and Treiman.

### 3.2 LEPTON PAIR LOCALITY THEOREMS AND THE SCALAR BOSON MEDIATED WEAK INTERACTION THEORIES.

The characteristic feature of a scalar boson mediated weak interaction model is the exchange of two particles to produce the lowest order weak process. Even to the lowest order in $G$, therefore, the lepton pair is not 'local' in the sense of Pais and Treiman, in that the lepton pair does not emerge from a single point in the Feynman diagram. A deviation from the explicit dependence on lepton pair variables as given by the Pais-Treiman theorem is expected in a scalar boson mediated theory, whereas such deviations would occur only to higher orders in a vector boson mediated theory or with the four fermion interaction. Whereas the higher order processes necessarily should become very important at energies of around 300 GeV, at lower energies of 30 - 40 GeV, the higher order processes are not a compelling necessity and may not be observable. Further the electromagnetic corrections which may also bring in such effects can occur to order $G\alpha$ and, therefore will be of the order of one per cent. Thus a test for the scalar boson mediated weak interaction theories would be to look for deviations from the explicit dependence on lepton variables as obtained from an assumption of locality, at intermediate energies

In the next section we evaluate violations of the energy theorem for the quasielastic process. In section 4, we estimate violations of the angle theorem for a simple inelastic reaction.

In section 5 we consider possibilities of such evaluations for other inelastic processes.

## 3.3 VIOLATION OF THE ENERGY DEPENDENCE THEOREM

We consider the quasielastic process

$$\bar{\nu}_\mu(q_1) + p(p_1) \to \mu^+(q_2) + n(p_2)$$

We calculate the amplitude for this process in a simplified version of the Gupta-Patil model in which the hadronic scalar and pseudoscalar currents are composed of the nucleon fields only and further consider only the lowest order diagrams.

The amplitude given by the box diagram of Figure 1 is

$$T = \frac{2g^4 \operatorname{Cos}\theta}{(2\pi)^4} \bar{v}(q_1)\, \gamma^\lambda\, (1-\gamma_5)\, v(q_2) \int \frac{d^4 q\, q_\lambda}{(q^2 - \mu^2)[(q-q_2)^2 - M^2][(q-L)^2 - M^2]}$$

$$\times\, \bar{u}_n(p_2)\, \frac{\{\gamma.(q+p_1-L) - m_N\}}{[(q + p_1 - L)^2 - m_N^2]}\, (1-\gamma_5)\, u_n(p_2)$$

where $\mu$ is the mass of the pseudoscalar meson and $m_N$ is the mass of the nucleon. In subsequent steps we retain only terms which would give rise to additional $M$ dependence to order $\frac{1}{M4}$. Thus

$$T = \frac{2g^4 \operatorname{Cos}\theta_c}{i(2\pi)^4} \bar{v}(q_1)\, \gamma^\lambda\, (1-\gamma_5) v(q_2)\, \bar{u}_n(p_2)\, \gamma^\sigma\, (1-\gamma_5)\, u_n(p_2)\, I_{\lambda\sigma}$$

where

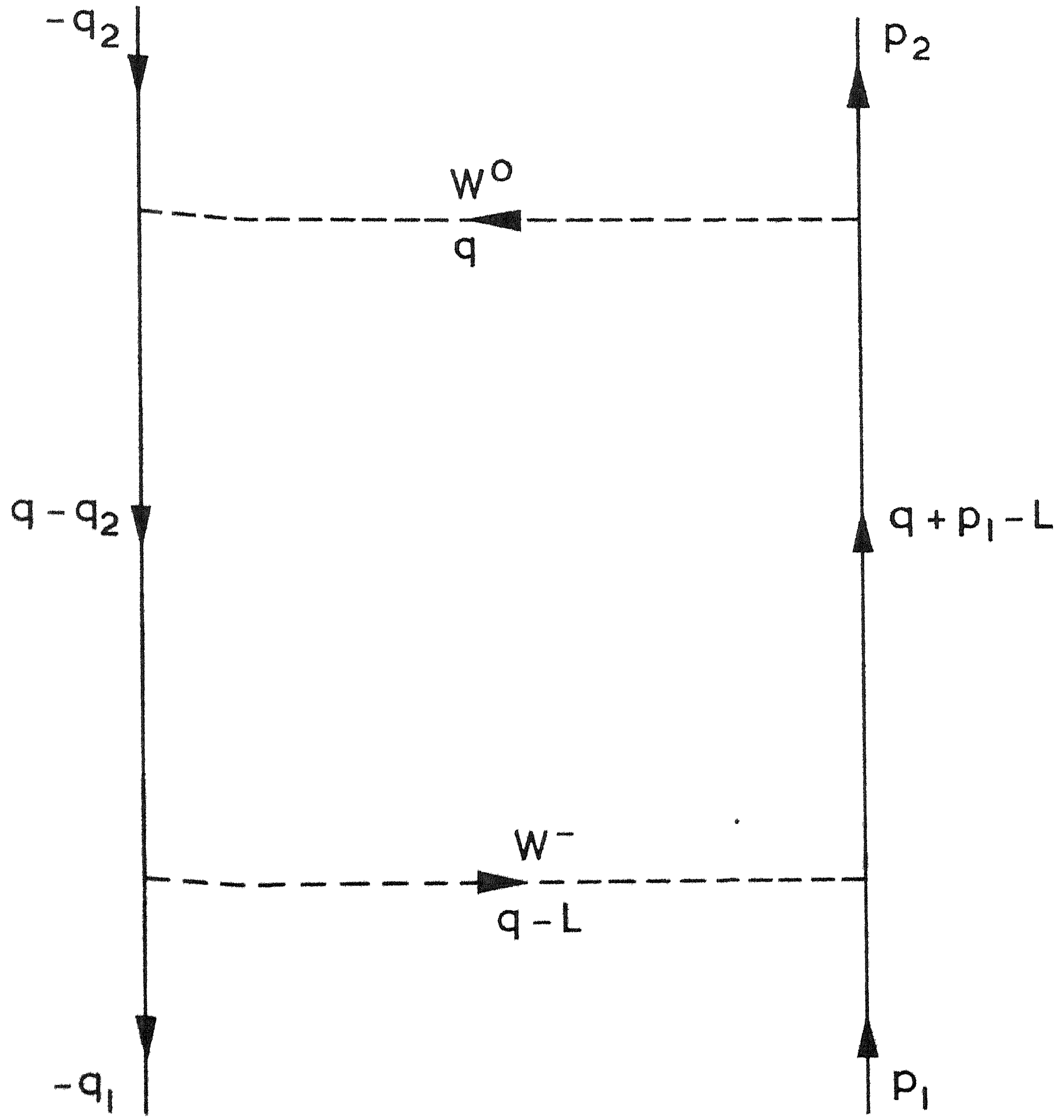


Figure 1

$$I_{\lambda\sigma} = \int \frac{d^4 q\, q_\lambda\, q_\sigma}{(q^2-\mu^2)\,[(q-q_2)^2 - M^2]\,[(q+p_1-L)^2 - r_N^2]\,[(q-L)^2 - M^2]}$$

$$= 6\int_0^1 x^2 y\, dx\, dy\, dz \int \frac{d^4 q\, q_\lambda\, q_\sigma}{(q^2+2q.p+s+i\varepsilon)^4}$$

$$q^2 + 2q.p + s = [(q-q_2)^2 - M^2]xyz + [(q-L)^2-M^2]xy(1-z)$$

$$+ (q^2-\mu^2)\, x(1-y) + [(q+p_1-L)^2-m_N^2](1-x)$$

$$= q^2+2q.[-q_2\, xyz + (p_1-L)(1-x)-Lxy(1-z)] - M^2 xy$$

$$+ txy(1-z) - \mu^2\, x(1-y) + [(p_1-L)^2-m_N^2](1-x)$$

$$\int \frac{d^4 q\, q_\sigma\, q_\lambda}{(q^2 + 2q.p+s+i\varepsilon)^4} = \frac{i\pi^2}{12}\left\{\frac{g_{\lambda\sigma}}{p^2-s} - \frac{2p_\sigma\, p_\lambda}{(p^2-s)^2}\right\}$$

$$p^2 - s = M^2 xy - 2q_2.p_1 xyz(1-x) + \text{other terms}$$

$$= M^2 xy - N.p_1\, x(1-x)yz + \text{other terms}$$

Coefficient of $g_{\lambda\sigma}$ in $I_{\lambda\sigma}$ is

$$\frac{i\pi^2}{2}\int_0^1 x^2 y\, dxdy\, dz\, \frac{1}{[M^2 xy - N.p_1\, x(1-x)yz + \text{other terms}]}$$

$$= \frac{i\pi^2}{2}\int_0^1 x^2 dxydydz\left[\frac{1}{M^2 xy} + \frac{N.p_1(1-x)z}{M^4 xy} + \text{other terms}\right]$$

$$= \frac{i\pi^2}{2}\cdot\frac{1}{M^2}\left[\frac{1}{2} + \frac{N.p_1}{12M^2}\right] \text{ retaining only terms of}$$

interest.

A more careful expansion is performed in appendix C.

Similarly only the $p_{1\lambda} N_\sigma$ term in $p_\lambda \; p_\sigma$ is retained in the second term.

$$I_{\lambda\sigma} = \frac{i\pi^2}{2} \cdot \frac{1}{2M^2} \left[ g_{\lambda\sigma} \left(1 + \frac{N.p_1}{6M^2}\right) + \frac{N_\sigma \, p_{1\lambda}}{6M^2} \right]$$

Substituting in T we obtain

$$T = \frac{G \cos \theta_c}{\sqrt{2}} \, l^\lambda \, J^\sigma \left[ g_{\lambda\sigma} \left(1 + \frac{N.p_1}{6M^2}\right) + \frac{N_\sigma \, p_{1\lambda}}{6M^2} \right]$$

where $l_\lambda = \bar{v}(q_1) \, \gamma_\lambda \, (1 - \gamma_5) \, v(q_2)$

$$J_\lambda = \bar{u}_n(p_2) \gamma_\lambda \, (1 - \gamma_5) \, u_n(p_1)$$

and $$\frac{g}{\sqrt{2}} = \frac{g^4}{(4\pi)^2} \; \frac{1}{2M^2}$$

$$\sum_{\text{spins}} |T|^2 = \frac{G^2 \cos^2 \theta_c}{2} \, l^\lambda \, (l^{\lambda'})^\dagger \, J^\sigma \, (J^{\sigma'})^\dagger \times$$

$$\left[ g_{\lambda\sigma} \, g_{\lambda'\sigma'} \left(1 + \frac{N.p_1}{3M^2}\right) + \frac{1}{6M^2}\left(g_{\lambda\sigma} \, N_{\sigma'} p_{1\lambda'} + g_{\lambda'\sigma'} N_\sigma \, p_{1\lambda}\right) \right]$$

The cross section is given for $t \ll M^2$ by, again retaining terms of the lowest order in $\dfrac{N.p_1}{M^2}$,

$$\left(\frac{d\sigma}{dt}\right)_{ST} = \left(1 + \frac{2N.p_1}{3M^2}\right) \left(\frac{d\sigma}{dt}\right)_{CC}$$

where $\left(\frac{d\sigma}{dt}\right)_{ST}$ is the differential cross section calculated in the scalar theory and $\left(\frac{d\sigma}{dt}\right)_{CC}$ the differential cross section

Table 1: Violation of the Theorem of Pais and Treiman in Scalar boson theories of Weak Interactions.

$$t = \text{Constant} << M^2$$

| Mass of the Scalar Boson | Laboratory Energy of the Neutrino in GeV | x |
|---|---|---|
| 10 GeV | 10 | 12.5 |
| | 20 | 25.0 |
| | 30 | 37.5 |
| | 50 | 62.5 |
| 15 GeV | 15 | 8.3 |
| | 30 | 16.6 |
| | 45 | 25.0 |
| | 60 | 33.2 |
| 20 GeV | 20 | 6.3 |
| | 50 | 15.7 |
| | 75 | 23.4 |
| | 100 | 31.5 |

in the current-current theory. Percentage violation of the theorem, x, defined by

$$x = \frac{\left[ \left( \frac{d\sigma}{dt} \right)_{ST} - \left( \frac{d\sigma}{dt} \right)_{CC} \right]}{\left( \frac{d\sigma}{dt} \right)_{CC}} \times 100$$

is shown for various energies and different masses of the scalar boson in Table 1. We notice that substantial violations of the locality theorem occur at energies much smaller than would be required for the production of the weak bosons. We would like to point out again that such large violations occur only in theories in which the lowest order weak process occurs through a fourth order interaction. Further, radiative corrections which would also induce similar effects are expected to be of the order of 1 per cent or less.

## 3.4 VIOLATION OF THE ANGLE THEOREM

The angle theorem is nonexistent for the quasielastic reaction, there being just two independent kinematic variables (E and t with our choice). We need at least one additional particle in the final state. To evaluate violations of the angle theorem we considered a simple illustrative example

$$\bar{\nu} + p \rightarrow n + \mu^{+} + \eta^{0} \quad .$$

Diagrams contributing to this process are shown in Figure 2. Diagram 2(a) does not contribute to the violation of the angle

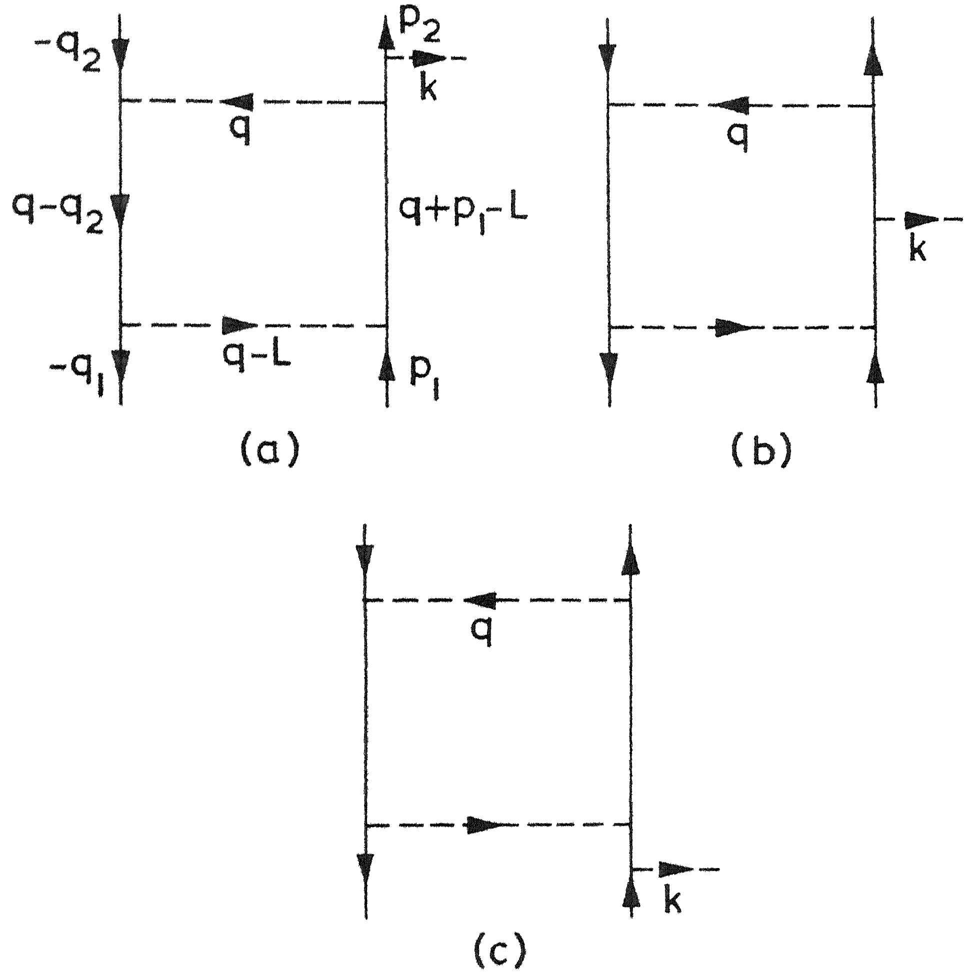


Figure 2

theorem, since in this case the only hadronic momentum occurring inside the q-integration box is $p_1$. Additional terms from this diagram, therefore, contain $N.p_1$ which is independent of the angle $\phi$. In the case of diagram 2(b), we find that terms involving $N$ occur only as coefficients of $1/M^6$ in the amplitude. The contribution of these terms to the violation of the angle theorem is, therefore, negligible. The diagram 2(c) does contribute to the violation of the angle theorem through terms like N.k. But in this case, at the energies and masses we have considered (Cf. Table 1), we find violation of the theorem to be of the order of 5 per cent or less (appendix D). Therefore, measurable violations of the angle theorem do not seem to occur in the scalar-boson-mediated theories.

## 3.5 INELASTIC REACTIONS

Inelastic processes involving more than one hadron in the final state do not, in principle, involve any substantial modifications. But actual calculation requires a specific model for the hadronic pseudoscalar and scalar currents coupling to the scalar boson. The violations, therefore, can be evaluated only in a specific model of the strong interactions. As an illustration we consider the single pion production reaction.

$$\nu_l(q_1) + p(p_1) \rightarrow p(p_2) + \pi^+(k) + l^-(q_2)$$

In this case in addition to diagrams similar to Figure 2, an additional diagram 3(a) has to be considered. This is demanded

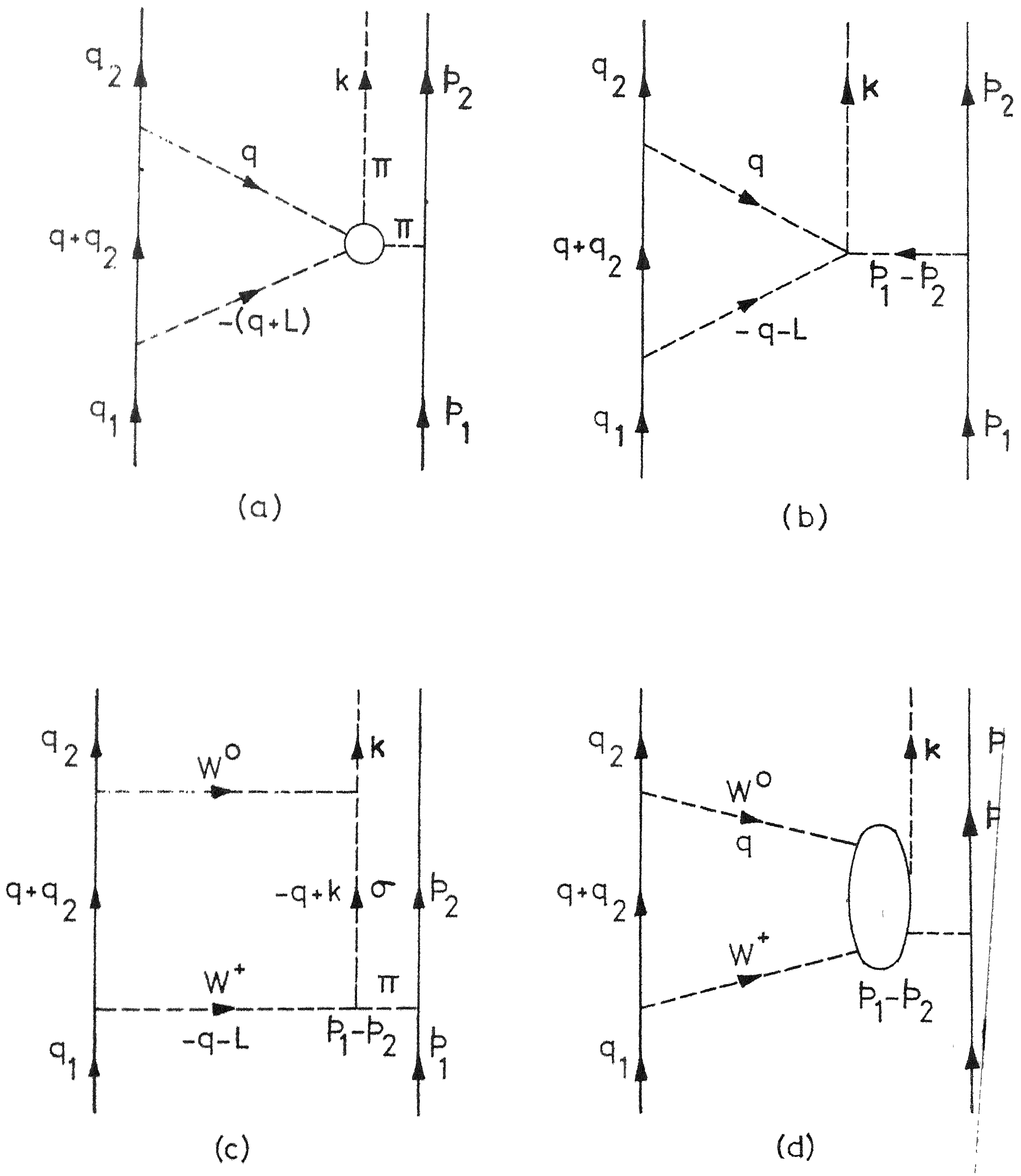
q 2
k
P 2
q
π
π
q+q 2
-(q+L)
q 1
P 1
(a)
q 2
k
P 2
q
q+q 2
P 1 - P 2
- q-L
q 1
P 1
(b)
q 2
W o
k
q+q 2
-q+k
σ
P 2
W +
π
-q-L
P 1-P 2
P 1
q 1
(c)
q 2
W o
k
q
q+q 2
W +
P 1-P 2
q 1
(d)

Figure 3

by the need to maintain CVC to each order in perturbation theory. But this diagram may or may not contribute additional terms depending on the structure of the $\pi^3 W$ vertex. A simple point vertex, as in Figure 3(b), leads to no hadron momenta in the $q$ integration box and, therefore, no additional $N$ terms after the $q$ integration is performed. Assuming a scalar particle coupling to $\pi^2$ and also $\pi W$, as in Figure 3(c), also leads to no additional $N$ dependence to the required order, since the integrals are more convergent and additional $N$ terms occur only to order $1/M^6$. On the other hand, if the hadronic pseudoscalar and scalar currents are constructed from Fermion fields only, diagrams like 3(d) will contribute additional terms containing $N$ to order $1/M^4$.

In the absence of a reasonably general and model independent approach, any further detailed calculation of inelastic reactions does not seem to be of interest for the present.

# CHAPTER - IV

## LEPTON ENERGY AND ANGLE DEPENDENCE THEOREMS FOR RADIATIVE NEUTRINO SCATTERING

In the last chapter, we considered the lepton pair locality theorems of Pais and Treiman and deviations from these results in scalar boson mediated weak interaction theories. Even when the local current-current form for the weak interaction Lagrangian is valid, 'non-local' violations of these theorems occur due to electromagnetic interactions. However, it can be shown that for processes involving emission of a single photon, which involve the weak and electromagnetic processes in the lowest order, explicit expressions for dependence of cross section on the lepton energy and angle variables can still be obtained as a consequence of the current-current form for the weak interaction Lagrangian. Expressions of Pais and Treiman for dependence on these variables are modified in a definite fashion[16].

### 4.2 KINEMATICS

Kinematics is similar to that of the Pais-Treiman theorems. We consider the process

$$\bar{\nu}_l(q_1) + p(p) \rightarrow l^+(q_2) + \alpha(P) + \gamma(k)$$

where $\alpha(P)$ is the final state hadronic system with total momentum $P$. We work throughout in the laboratory frame $(\vec{p}=o)$

and take mass of the lepton equal to zero. As before, we define $L = q_2-q_1$ and $N = q_2+q_1$. We choose the z direction along $\vec{L}$, the y-axis along $\vec{L}x\vec{k}$ and the x-axis along $(\vec{L}x\vec{k})x\vec{L}$. Further, let $\Theta$ be the angle between k and the z-axis and $\phi$ between the x-axis and the projection of $\vec{N}$ on the xy plane. If there are n particles in the system $\alpha$, we will need in all 3n+2 variables to describe the scattering process. Some of them are chosen to be

1. E, the laboratory energy of the incident neutrino
2. The angle $\phi$ as defined above
3. $t = (q_2-q_1)^2$, the square of the momentum transfer from the leptons
4. $w = P^2$ square of the invariant mass of the final state hadronic complex
5. $s = (k + P)^2$ and
6. $k_o$ the energy of the emitted photon.

The remaining 3n-4 variables are internal to the system $\alpha$ and do not concern us here.

The components of N are given by similar expressions

$$N_o = 2E + L_o = 2E + \frac{1}{2m_N}(m_N^2 + t-s)$$

$$N_z = \frac{1}{m_N(Y^2 - tm_N^2)^{1/2}}\left[ Y^2 + 2tm_N \right]$$

$$N_y = n \operatorname{Sin} \phi \qquad N_x = n \operatorname{Cos} \phi \qquad \text{where } n^2 = t\left(1 - \frac{N_o^2}{L_z^2}\right).$$

## 4.3 IMPLICATIONS OF THE LOCAL CURRENT-CURRENT LAGRANGIAN FOR RADIATIVE NEUTRINO SCATTERING

The amplitudes for the process are

$$T_{fi}^{(1)} = -\frac{eG}{\sqrt{2}} \varepsilon^{\mu} \bar{v}(q_1) \gamma^{\sigma} (1-\gamma_5) \frac{1}{\gamma\cdot(k+q_2)+m_1} \gamma_{\mu} v(q_2)\langle\alpha|J_{\sigma}^{W}|p\rangle$$

and

$$T_{fi}^{(2)} = \frac{eG}{\sqrt{2}} \varepsilon^{\mu} \bar{v}(q_1) \gamma^{\sigma} (1-\gamma_5) v(q_2) M_{\mu\sigma}$$

$$\text{where} \quad M_{\mu\sigma} = i \int d^4x\, e^{ik.x} \langle\alpha|T^{*}[\, J_{\mu}^{em}(x)\, J_{\sigma}^{W}(0)\,]\,|p\rangle$$

Gauge invariance of the sum of these can be checked. On replacing $\varepsilon^{\mu}$ by $k^{\mu}$, $T_{fi}^{(1)}$ becomes $-\frac{eG}{\sqrt{2}} \bar{v}(q_1) \gamma^{\sigma} (1-\gamma_5) v(q_2)\langle\alpha|J_{\sigma}^{W}|p\rangle$ and $T_{fi}^{(2)}$ becomes $\frac{eG}{\sqrt{2}} \bar{v}(q_1) \gamma^{\sigma} (1-\gamma_5) v(q_2) \langle\alpha|J_{\sigma}^{W}|p\rangle$ since

$$k^{\mu} M_{\mu\sigma} = -\langle\alpha|\int d^3x\, e^{-ik.x} [\, J_{0}^{em}(\vec{x},o),\, J_{\sigma}^{W}(0)\,]\,|p\rangle$$

$$+ \text{ divergence of sea gull term}$$

$$= \langle\alpha\,|\,J_{\sigma}^{W}(0)\,|\,p\rangle\,.$$

Squaring the amplitude and summing over spins, we have

$$\Sigma|T|^2 = \left[\, \tau_1^{\lambda\sigma}\, T_{\sigma\lambda}^{1} + \tau_2^{\lambda\sigma}\, T_{\sigma\lambda}^{2} + (\, \tau^{\lambda\sigma\nu}\, T_{\nu\sigma\lambda} + h.c)\,\right] \frac{G^2e^2}{m_1 q_{10}} \qquad \ldots(1)$$

where

$$\tau_{1\lambda\sigma} = \frac{1}{k.q_2} (q_{1\lambda}\, k_{\sigma} + k_{\lambda} q_{1\sigma} - k.q_1 g_{\lambda\sigma} + i\varepsilon_{\alpha\beta\lambda\sigma}\, k^{\alpha}\, q_1^{\beta}\,),$$

$$\tau_{2\lambda\sigma} = -(q_{1\lambda}\, q_{2\sigma} + q_{2\lambda}\, q_{1\sigma} - q_1\cdot q_2 g_{\lambda\sigma} + i\varepsilon_{\alpha\beta\lambda\sigma}\, q_2^\alpha\, q_1^\beta\,),$$

$$\begin{aligned}\tau_{\lambda\sigma\nu} = \frac{1}{k\cdot q_2} [\, & 2q_{2\nu}\,(q_{1\lambda}q_{2\sigma} + q_{2\lambda}q_{1\sigma} - q_1\cdot q_2 g_{\lambda\sigma} + i\varepsilon_{\alpha\beta\lambda\sigma}\, q_2^\alpha\, q_1^\beta), \\ & + q_{1\lambda}(k_\sigma q_{2\nu} - k\cdot q_2 g_{\sigma\nu} + k_\nu\, q_{2\sigma}) - k\cdot q_1(g_{\nu\lambda}q_{2\sigma} - q_{2\lambda}g_{\nu\sigma} \\ & + q_{2\nu}g_{\lambda\sigma}) + q_{1\nu}(k_\lambda\, q_{2\sigma} + k\cdot q_2 g_{\lambda\sigma} - q_{2\lambda}\, k_\sigma) \\ & - q_1\cdot q_2(k_\lambda g_{\nu\sigma} - k_\sigma g_{\nu\lambda} + k_\nu\, g_{\lambda\sigma}) + q_{1\sigma}\,(k_\lambda\, q_{2\nu} \\ & - k\cdot q_2 g_{\nu\lambda} + k_\nu q_{2\lambda}) + i\,(q_{2\sigma}\,\varepsilon_{\alpha\beta\nu\lambda}q_1^\alpha k^\beta + \varepsilon_{\alpha\beta\gamma\lambda}\, q_1^\alpha\, q_2^\beta\, k^\gamma g_{\nu\sigma} \\ & + q_{2\nu}\,\varepsilon_{\alpha\beta\lambda\sigma}k^\alpha q_1^\beta + k_\sigma\,\varepsilon_{\alpha\beta\lambda\nu}\, q_1^\alpha\, q_2^\beta + k\cdot q_2\,\varepsilon_{\alpha\nu\lambda\sigma}\, q_1^\alpha \\ & + k_\nu\,\varepsilon_{\alpha\beta\sigma\lambda}\, q_1^\alpha q_2^\beta + g_{\lambda\sigma}\,\varepsilon_{\alpha\beta\gamma\nu}\, q_1^\alpha k^\beta q_2^\gamma + q_{2\lambda}\,\varepsilon_{\alpha\beta\nu\sigma} \\ & q_1^\alpha\, k^\beta + g_{\nu\lambda}\varepsilon_{\alpha\beta\gamma\sigma}\, q_1^\alpha\, q_2^\beta\, k^\gamma + k_\lambda\,\varepsilon_{\alpha\beta\nu\sigma}\, q_2^\alpha\, q_1^\beta \\ & + q_1\cdot q_2\,\varepsilon_{\alpha\lambda\nu\sigma}k^\alpha + q_{1\sigma}\,\varepsilon_{\alpha\beta\nu\lambda}k^\alpha q_2^\beta + q_{1\nu}\,\varepsilon_{\alpha\beta\lambda\sigma}k^\alpha q_2^\beta \\ & + k\cdot q_1\,\varepsilon_{\alpha\nu\lambda\sigma}q_2^\alpha + q_{1\lambda}\,\varepsilon_{\alpha\beta\nu\sigma}\, q_2^\alpha\, k^\beta\,)\,],\end{aligned}$$

$$T^1_{\lambda\sigma} = \sum_{\text{spins}} \langle\, \alpha|J^W_\lambda(0)|p\rangle\, \langle\alpha|J^W_\sigma(0)|p\rangle^\dagger \;,$$

$$T^2_{\lambda\sigma} = M_{\mu\lambda}\,(M_{\mu'\sigma'})^\dagger\;\; g^{\mu\mu'} \;,$$

and

$$T_{\nu\sigma\lambda} = (M_{\nu\sigma}\;)^\dagger\, \langle\alpha|J^W_\lambda(0)|p\rangle\;.$$

Now, $\langle\alpha|J^W_\sigma(0)|p\rangle$ and $M_{\mu\sigma}$ are functions of the momenta of the photon and the hadrons in $\alpha$ and depend on the lepton

momenta only through the constraints imposed by four momentum conservation. These, therefore, are functions of $L$ but not $N$ which implies they are independent of the two lepton variables $E$ and $\phi$ since $N$ as before has two constraints $N^2 = -t$ and $N \cdot L = 0$. The dependence of cross sections on $E$ and $\phi$, therefore, come through the lepton tensors $\tau_{\lambda\sigma}{}^{1,2}$ and $\tau_{\lambda\sigma\nu}$.

Integrating (1) over the variables of the hadronic system and also $\phi$, the differential cross section is given as a function of $E$ by (please see appendix C for details)

$$\frac{d\sigma}{dsdtdwdk_o} = \frac{1}{\sqrt{(E-E')^2-t}} \cdot \frac{1}{E}\left[AE^2+BE+C+\frac{A_1E^3+B_1E^2+C_1E+D_1}{(PE^2+QE+R)^{1/2}}\right] \qquad \ldots (2)$$

P, Q and R are given by

$$P = m_N t \sin^2\theta + (X-Y\cos\theta)^2$$

$$Q = \frac{1}{m_N}\left[m_N^2 t \sin^2\theta\, Y + (X-Y\cos\theta)\,(2XY - (X^2+Y^2)\cos\theta)\right]$$

$$R = \frac{1}{4m_N^2}\left[m_N^4 t^2 \sin^2\theta + (2XY - (X^2+Y^2)\cos\theta)^2\right]$$

where $Y = \frac{1}{2}(m_N^2 + t-s)$ and $X = (Y^2 - m_N^2 t)^{1/2}$

If integrations are performed only over the variables internal to the hadronic system $\alpha$, we have the $\phi$ dependence theorem

$$\frac{d\sigma}{ds\,dt\,dw\,dk_o\,d\phi} =$$

$$\frac{1}{E[(E-E')^2-t]^{1/2}}\Big[a \cos 2\phi + b \sin 2\phi + c \cos \phi + d \sin \phi + e +$$

$$\frac{a_1 \cos 3\phi + b_1 \sin 3\phi + c_1 \cos 2\phi + d_1 \sin 2\phi + e_1 \cos \phi + f_1 \sin \phi + g_1}{(q + r \cos \phi)}\Big]$$

...(3)

$a$, $b$, $a_1$, $b_1$, etc., are functions of $s$, $t$, $w$, $k_o$ and $E$. $q$ and $r$ are given by

$$q = 2E(X - Y \cos \theta) + \frac{1}{m_N}\left[2XY - (X^2 + Y^2) \cos \theta\right]$$

$$r = \sin \theta \left[(-tm_N)(4m_N^2 E^2 + 4YE + m_N t)\right]^{1/2}$$

Expressions (2) and (3) have been obtained assuming a local current-current form for the weak interaction Lagrangian. These results also hold in vector boson mediated theories of weak interactions. Even though the lepton pair in a radiative process is not 'local', the non-locality is of a specific form since the electromagnetic interactions of the charged leptons are known. By an appropriate choice of kinematic variables, therefore, the dependence on the lepton energy and angle can be made explicit.

# CHAPTER - V

## VIOLATION OF LEPTON PAIR LOCALITY THEOREMS FOR RADIATIVE NEUTRINO SCATTERING IN SCALAR THEORIES OF WEAK INTERACTIONS.

In contrast to violations of the Pais and Treiman theorems for neutrino reactions, violations of the theorems for radiative neutrino reactions derived in the previous chapter can be calculated in scalar boson mediated models of weak interactions independent of the details of the weak currents of hadrons, retaining only some asymptotic expression for the product of the hadronic currents. This is possible essentially because the photon can couple to the charged lines of the box diagram typical of the scalar models. In our calculations we assume the box integral receives contribution mainly from the high energy region and retain the Bjorken limit for the product of the hadronic currents. We can thus obtain additional momentum dependence arising from a scalar boson mediated weak interaction structure for a radiative neutrino scattering process with an arbitrary hadronic system in the final state in terms of matrix elements of physical vector and axial vector currents. This constitutes section 2. In section 3 we obtain numerical values for the reaction with only a nucleon as the final state hadron[17].

## 5.2 THE REACTION $\bar{\nu} + p \rightarrow \alpha + \mu^+ + \gamma$

Diagrams contributing to the process

$$\bar{\nu}(q_1) + p(p_1) \to \alpha(p) + \mu^+(q_2) + \gamma(k)$$

where $\alpha(p)$ is an arbitrary hadronic system are shown in Figure 4. We consider first Figure 4(a). The amplitude is

$$T_{fi}^{(1)} = -eg^4 \varepsilon^\mu \int \frac{d^4q}{(2\pi)^4} \bar{v}(q_1)(1+\gamma_5) \frac{\gamma\cdot(q-q_2-k)+M}{(q-q_2-k)^2-M^2} \gamma_\mu \frac{\gamma\cdot(q-q_2)+M}{(q-q_2)^2-M^2}$$

$$\times (1-\gamma_5) v(q_2) \frac{1}{(q^2-\mu^2)} \frac{1}{[(q-k-L)^2-M^2]}$$

$$\times \int d^4x\, e^{iq\cdot x} \langle \alpha |T[P_1^1(x)\, j^-(0)]| p\rangle \qquad \ldots(1)$$

where $\quad j^- = (S_1^2 - P_1^2) \cos\theta_c + (S_1^3 - P_1^3) \sin\theta_c.$

Assuming the contribution to $q$ integration comes from high $q_0$ values we retain the Bjorken limit for the time ordered product. For equal time commutators of the hadronic scalar and pseudoscalar currents, the quark model commutation relations are assumed to hold in the Gupta-Patil model. Thus

$$\int d^4x\, e^{iq\cdot x} \langle\alpha|T[P_1^1(x)\, j^-(0)]|p\rangle \to \frac{iq^\mu}{q^2} \langle\alpha|J_\mu^-(0)|p\rangle$$

where $\quad J_\mu^- = (V_{1\mu}^2 - A_{1\mu}^2) \cos\theta_c + (V_{1\mu}^3 - A_{1\mu}^3) \sin\theta_c$

The box integral in (1) can now be performed. Retaining the terms to order $1/M^4$, we have

$$T_{fi}^{(1)} = \frac{Gc}{\sqrt{2}} \varepsilon^\mu \left(\frac{7}{18M^2}\right) \bar{v}(q_1)\, \gamma^\lambda(1-\gamma_5)\, v(q_2) \langle\alpha|J^{\sigma -}|p\rangle$$

$$\times (N_\mu\, g_{\lambda\sigma} + N_\sigma\, g_{\mu\lambda}).$$

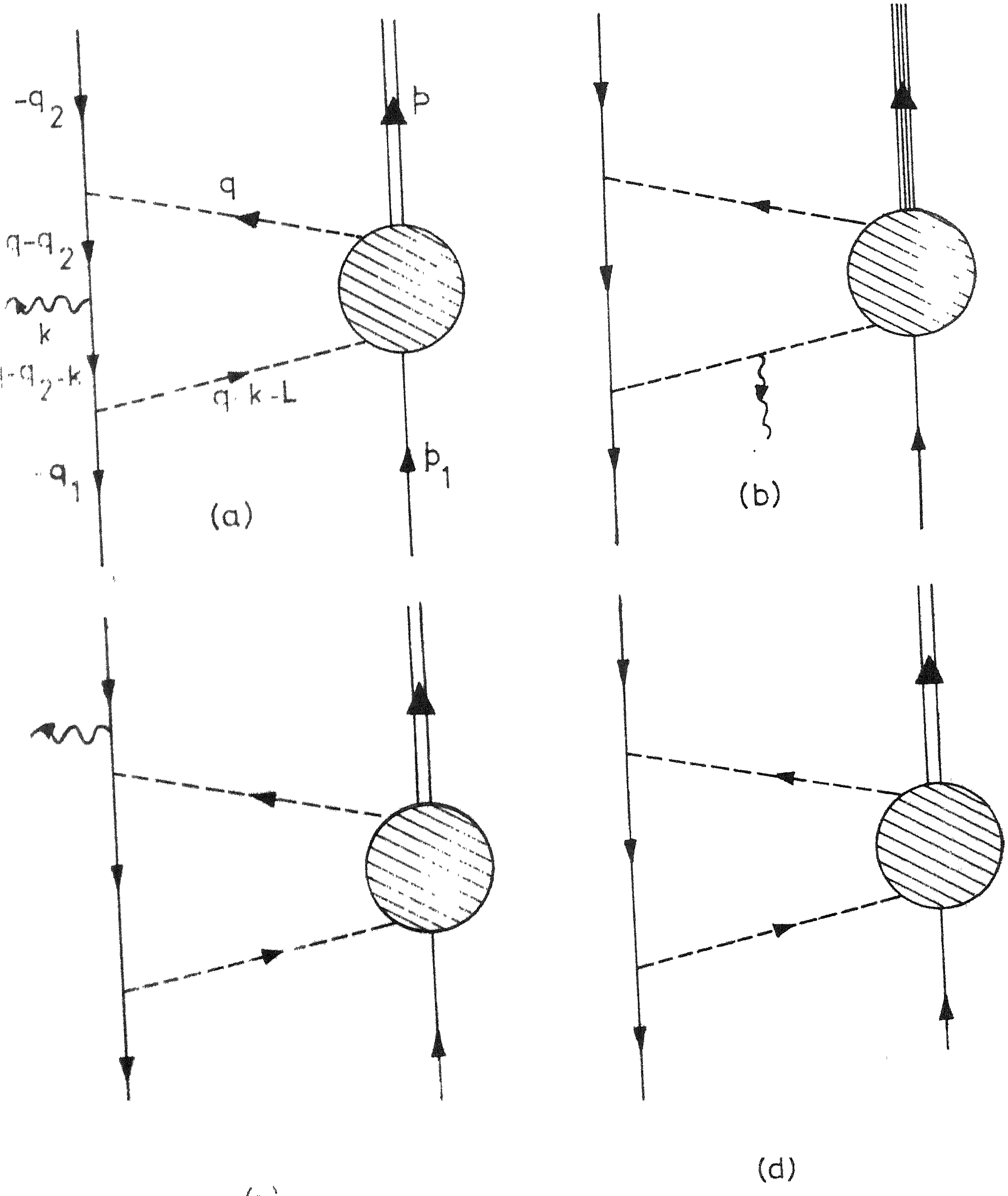


Figure 4

Amplitudes corresponding to the other diagrams in Figure 4 are similarly evaluated. The corresponding amplitudes are

$$T_{fi}^{(2)} = -\frac{Ge}{\sqrt{2}} \cdot \frac{1}{12M^2} \epsilon^{\mu} l^{\lambda} \langle\alpha|J^{\sigma-}|p\rangle (g_{\mu\lambda} N_{\sigma} + g_{\lambda\sigma} N_{\mu})$$

$$T_{fi}^{(3)} = -\frac{Ge}{\sqrt{2}} \epsilon^{\mu}\Big[\frac{i}{2M^2} l_{\mu} N^{\sigma} \langle\alpha|J_{\sigma}^{-}|p\rangle + \frac{1}{2k.q_2}\bar{v}(q_1)\gamma^{\lambda}\gamma\cdot(k+q_2)$$

$$\times \gamma_{\mu}(1-\gamma_5)v(q_2)\langle\alpha|J^{\sigma-}|p\rangle(g_{\lambda\sigma}(1+\frac{k.N}{3M^2}) - \frac{k_{\lambda} N_{\sigma}}{6M^2})\Big]$$

and $$T_{fi}^{(4)} = \frac{Ge}{\sqrt{2}} \epsilon^{\mu} l^{\lambda} M_{\mu\lambda}$$

where $$M_{\mu\lambda} = i \int d^4x \langle\alpha|T(J_{\mu}^{em}(x) J_{\lambda}^{-}(0))|p\rangle .$$

For any specific hadronic system $\alpha(p)$, explicit expressions for $\langle\alpha|J_{\lambda}^{-}|p\rangle$ and $M_{\mu\lambda}$ are needed to evaluate the non-local effects. These are the experimentally obtainable matrix elements of the physical vector and axial vector currents of the hadrons.

We have estimated the contribution of additional lepton momentum terms for the process $\bar{\nu} + p \rightarrow n + \mu^{+} + \gamma$.

## 5.3 NUMERICAL VALUES FOR $\bar{\nu} + p \rightarrow n + \mu^{+} + \gamma$

The matrix elements needed for this case are $M_{\mu\lambda}$ and

$$\langle n|J_{\lambda}^{1-i2}(0)|p\rangle = \bar{u}(p_2)\, \Gamma_{\lambda}(p_2,p_1)\, u(p_1)$$

where

$$\Gamma_\lambda(p_2,p_1) = \gamma_\lambda \; F_1^V(r^2) + i\,\sigma_{\lambda\nu}\,(p_2-p_1)^\nu \; F_2^V(r^2) + \gamma_\lambda\gamma_5 \; F_A(r^2)$$
$$+ (p_2-p_1)_\lambda \; \gamma_5 \; F_P(r^2)$$

with $r = p_2-p_1$ .

For $M_{\mu\lambda}$ we consider only the insertion of the nucleon intermediate states between the weak and electromagnetic currents. It is, therefore, given by

$$M_{\mu\lambda} = -\bar{u}(p_2)\left[\Gamma_\lambda\,(p_2,p_1-k)\,\frac{1}{\gamma\cdot(p_1-k)-m_N}\,(\gamma_\mu+\frac{i\,\mu_P}{2m_N}\,\sigma_{\mu\nu}k^\nu\,)\right.$$
$$\left.+\frac{i\,\mu_n}{2m_N}\,\sigma_{\mu\nu}\,k^\nu\;\frac{1}{\gamma\cdot(p_2+k)-m_N}\,\Gamma_\lambda(p_2+k,p_1)\right]u(p_1)$$

In numerical evaluations, we have used the following expressions for the form factors (Cf. Llewellyn Smith, Ref. 1).

1. The Dirac electromagnetic isovector form factor

$$F_1^V\,(r^2) = F_1^{\,P}(r^2) - F_1^{\,n}\,(r^2)$$
$$= (1-\frac{r^2}{4m_N^{\;2}})^{-1}\left[G_E^V\,(r^2) - \frac{r^2}{4m_N^{\;2}}\,G_M^V\,(r^2)\right]$$

The Sachs form factors are described experimentally to within 10 per cent by a dipole fit.

$$G_E^V(r^2) = (1-\frac{r^2}{0.71\mathrm{GeV}^2})^{-2}$$

$$G_M^V(r^2) = (1+\mu_P-\mu_n)\,(1-\frac{r^2}{0.71\mathrm{GeV}^2})^{-2}$$

2. The Pauli electromagnetic isovector form factor $F_2^V$

$$F_2^V = \frac{\mu_P F_2^P(r^2) - \mu_n F_2^n(r^2)}{\mu_P - \mu_n} = (\mu_P - \mu_n)^{-1} \left(1 - \frac{r^2}{4m_N^2}\right)^{-1}$$

$$\times \left[ G_M^V(r^2) - G_E^V(r^2) \right]$$

3. The axial form factor $F_A$

$$F_A(r^2) = -1.23 \left[ 1 - \frac{r^2}{(1.34\text{GeV})^2} \right]^{-2}$$

4. The Induced pseudoscalar form factor

$$F_P(r^2) = \frac{2m_N^2 \; F_A(r^2)}{(m_N^2 - r^2)}$$

## 5.4 VIOLATION OF THE ENERGY DEPENDENCE THEOREM FOR RADIATIVE SCATTERING

Additional terms involving higher powers of the neutrino energy are added to the results derived assuming a local current-current weak Lagrangian. Keeping only terms to order $1/M^2$ where $M$ is the mass of the scalar boson, the expression for energy dependence is modified to

$$\frac{d\sigma}{dsdtdwdk_o} = \frac{1}{\sqrt{(E-E')^2 - t}} \cdot \frac{1}{E} \left[ A_S E^3 + B_S E^2 + C_S E + D_S + \frac{A_{1S} E^4 + B_{1S} E^3 + C_{1S} E^2 + D_{1S} E + E_{1S}}{(PE^2 + QE + R)^{1/2}} \right]$$

For reaction with a nucleon only as the final state hadron, $w = m_N^2$. To estimate violation of the theorem we

evaluated numerically the ratio of the $A_S E^3$ term to the differential cross section $\left(\frac{d\sigma}{dsdtdk_o}\right)_{CC}$ in the current-current theory for various values of $s$, $t$ and $k_o$ in the permissible kinematic range. The latter cross section $\left(\frac{d\sigma}{dsdtdk_o}\right)_{CC}$ was calculated with only point hadronic weak vertices. We found substantial values for the $E^3$ term for values of energy such that $E < \frac{1}{4m_N^2}M^2$ where $M$ is the mass of the scalar boson. The ratio takes values of 0.6 or more in certain kinematic regions, generally smaller values of $s$ and $t$ and larger values of $k_o$. Some numerical values are quoted in Table 2.

A complete calculation would involve evaluation of the contribution of each term involving powers of $E$ to the cross section and comparing it with the value of the $E^3$ term expected from the scalar theory. The arithmetical labour involved in such a calculation is tremendous and does not seem to be of interest at the present stage.

## 5.5 VIOLATION OF THE ANGLE DEPENDENCE THEOREM FOR RADIATIVE SCATTERING

Terms involving functions of higher multiples of $\phi$ are added to the expressions derived earlier assuming a local current Lagrangian. For the reaction $\bar{\nu} + p \rightarrow n + \mu^+ + \gamma$ with the kinematics we have chosen these involve cosine functions of higher multiples of $\phi$ (dependence on $\phi$ occurs through the scalar product $N.k$ which involves only $\cos\phi$ ).

However, in numerical evaluations we did not find any large contributions from coefficients of terms involving larger multiples of $\phi$. The coefficient of the additional Cos $3\phi$ term, for example, brings in a multiple $\frac{nk_o}{M^2}$. In the region of not too large momentum transfers to which we restricted ourselves, this factor does not become large enough for the Cos $3\phi$ term to be important.

Table 2: Violation of lepton pair locality theorems for the process $\bar{\nu} + p \rightarrow n + \mu^{+} + \gamma$

$$x = A_S E^3 \times 100 \;/\; \left( \frac{d\sigma}{ds\,dt\,dk_o} \right)_{CC}$$

Values for the photon energy $k_o$ are chosen to be at the middle of the permitted range with the specified values of s and t

(1) Mass of the scalar boson M = 15 GeV

E = 30 GeV

| s in GeV$^2$/t in GeV$^2$ | | -1 | -2 | -3 | -4 | -5 |
|---|---|---|---|---|---|---|
| 17 | $k_o$ in GeV | 4.8 | 5.0 | 5.3 | 5.5 | 5.8 |
| | x | 5.41 | 0.70 | 0.04 | 0.04 | 0.01 |
| 20 | $k_o$ in GeV | 5.6 | 5.8 | 6.1 | 6.3 | 6.6 |
| | x | 6.35 | 0.82 | 0.18 | 0.05 | 0.02 |
| 25 | $k_o$ in GeV | 6.9 | 7.2 | 7.4 | 7.7 | 7.9 |
| | x | 7.80 | 1.01 | 0.22 | 0.07 | 0.02 |

E = 50 GeV

| s in GeV$^2$/t in GeV$^2$ | | -1 | -2 | -3 | -4 | -5 |
|---|---|---|---|---|---|---|
| 25 | $k_o$ in GeV | 6.9 | 7.2 | 7.4 | 7.7 | 7.9 |
| | x | 14.23 | 1.99 | 0.48 | 0.15 | 0.06 |
| 50 | $k_o$ in GeV | 13.6 | 13.8 | 14.1 | 14.3 | 14.6 |
| | x | 27.07 | 3.73 | 0.89 | 0.28 | 0.11 |

Table 2 (contd.)

| | | | | | | |
|---|---|---|---|---|---|---|
| 60 | $k_o$ in GeV | 16.2 | 16.5 | 16.7 | 17.0 | 17.3 |
| | x | 29.20 | 3.94 | 0.92 | 0.29 | 0.11 |
| 75 | $k_o$ in GeV | 20.2 | 20.5 | 20.7 | 21.0 | 21.3 |
| | x | 22.15 | 2.82 | 0.61 | 0.17 | 0.06 |
| 80 | $k_o$ in GeV | 21.5 | 21.8 | 22.1 | 22.3 | 22.6 |
| | x | 16.64 | 2.02 | 0.41 | 0.10 | 0.03 |

(2) <u>Mass of the scalar boson M = 20 GeV</u>

E = 50 GeV

| s in GeV$^2$/t in GeV$^2$ | | -1 | -2 | -3 | -4 | -5 |
|---|---|---|---|---|---|---|
| 50 | $k_o$ in GeV | 13.6 | 13.8 | 14.1 | 14.3 | 14.6 |
| | x | 15.23 | 2.10 | 0.50 | 0.16 | 0.06 |
| 60 | $k_o$ in GeV | 16.2 | 16.5 | 16.7 | 17.0 | 17.3 |
| | x | 16.42 | 2.22 | 0.52 | 0.16 | 0.06 |
| 75 | $k_o$ in GeV | 20.2 | 20.5 | 20.7 | 21.0 | 21.3 |
| | x | 12.46 | 1.59 | 0.34 | 0.10 | 0.03 |

E = 75 GeV

| s in GeV$^2$/t in GeV$^2$ | | -1 | -2 | -3 | -4 | -5 |
|---|---|---|---|---|---|---|
| 25 | $k_o$ in GeV | 6.9 | 7.2 | 7.4 | 7.7 | 7.9 |
| | x | 12.42 | 1.78 | 0.44 | 0.14 | 0.06 |

Table 2 (contd.)

| | | | | | |
|---|---|---|---|---|---|
| 50 | $k_o$ in GeV | 13.6 | 13.8 | 14.1 | 14.3 |
| | x | 24.61 | 3.58 | 0.90 | 0.30 |
| 60 | $k_o$ in GeV | 16.2 | 15.5 | 16.7 | 17.0 |
| | x | 29.38 | 4.26 | 1.07 | 0.36 |
| 75 | $k_o$ in GeV | 20.2 | 20.5 | 20.7 | 21.0 |
| | x | 35.65 | 5.11 | 1.27 | 0.42 |
| 80 | $k_o$ in GeV | 21.5 | 21.8 | 22.1 | 22.3 |
| | x | 37.2 | 5.31 | 1.32 | 0.43 |
| 90 | $k_o$ in GeV | 24.2 | 24.5 | 24.7 | 25.0 |
| | x | 38.76 | 5.48 | 1.34 | 0.44 |

E = 100 GeV

| s in $GeV^2$/t in $GeV^2$ | | -1 | -2 | -3 | -4 |
|---|---|---|---|---|---|
| 90 | $k_o$ in GeV | 24.2 | 24.5 | 24.7 | 25.0 |
| | x | 59.20 | 8.70 | 2.21 | 0.75 |
| 100 | $k_o$ in GeV | 26.9 | 27.1 | 27.4 | 27.6 |
| | x | 64.62 | 9.45 | 2.39 | 0.81 |
| 125 | $k_o$ in GeV | 33.5 | 33.8 | 34.0 | 34.3 |
| | x | 70.35 | 10.14 | 2.53 | 0.84 |
| 150 | $k_o$ in GeV | 40.2 | 40.4 | 40.7 | 40.9 |
| | x | 54.46 | 7.67 | 7.86 | 0.60 |

# CHAPTER - VI

## CONCLUSION

In this final chapter we summarize our results and also comment on some topics of related interest. In section 2, we comment on the possibility of calculating violations of lepton pair locality theorems in the gauge theory formulation of the vector boson mediated model of weak interactions. Violations of lepton pair locality theorems have also been considered in connection with the possible weak resonances and heavy leptons. These are discussed in section 3.

In the preceding chapters our emphasis has been on the importance of lepton pair locality theorems in the attempt to build a complete theory of weak interactions. Specifically, we were concerned with models of weak interactions which involve exchange of two particles for the first order weak processes and exhibit violations of the lepton pair locality theorems even in the comparatively lower energy range in which only the lowest order weak process may dominate. For the quasielastic reaction with a nucleon as the only final state hadron we found that the additional term with a higher power of energy can contribute 30 - 40 per cent or more to the cross sections expected from the current-current theory at energies much smaller than required for the production of the boson (Table 1). On the other hand, violations of the angle

dependence theorem due to kinematic restrictions seem to be very small.

As far as our results are concerned, these are to be considered as estimates indicative of the magnitude of the effects. This is because in calculating the semileptonic reactions, we had to develop some kind of approximation procedure for the hadronic part. For the quasielastic neutrino scattering process we thus retained only the lowest order diagram and for the radiative neutrino scattering we assumed the integrals received contribution mainly from the high momentum region. As far as the neutrino reactions with more than one hadron in the final state were concerned we were unable to evolve a satisfactory technique for evaluating the violations.

As regards the radiative neutrino reactions, investigations may not be possible in the first generation of neutrino experiments since the amplitudes are of order Ge. But in addition to the interest in regard to our present study, this reaction will be of considerable importance on its own. The radiative process is the simplest after the lowest order weak process and the usefulness of the photon in probing the vertices is considerable. The theorems concerning radiative neutrino reactions, therefore, are of considerable potential use beyond the one they have been put

to in our investigation. The latter remark is in order since as a test for the scalar boson mediated theories violations of the Pais and Treiman theorems will be presumably more easily studied than violations of the theorems for radiative scattering.

## 6.2 LEPTON PAIR LOCALITY THEOREMS AND THE GAUGE THEORIES OF WEAK INTERACTIONS

At present considerable interest and enthusiasm have been generated by the discovery that spontaneously broken gauge theories of the Yang-Mills type are renormalizable. In addition to providing a well-behaved theory of weak interactions, these theories offer an elegant unification of weak and electromagnetic interactions[5].

As far as the topic of our immediate interest is concerned, in these theories the lowest order weak processes are due to the exchange of a single vector boson and no violations of the lepton pair locality theorems of Pais and Treiman to order $G$ or of the theorems considered by us for radiative neutrino scattering to order $Ge$ occur. Violations of these theorems occur in higher orders due to the exchange of an additional neutral boson or a photon. Therefore, violations of these theorems are expected to be small. However, the Gauge theories provide a framework for calculating higher order contributions which could not be meaningfully evaluated in a simple minded vector boson theory.

## 6.3 VIOLATIONS OF LEPTON PAIR LOCALITY THEOREMS DUE TO WEAK RESONANCES AND HEAVY LEPTONS

C.H. Albright and R.H. Good Jr.[19] have considered resonance formation in high energy neutrino scattering. If a neutrino-nucleon resonance is formed in the reaction

$$\nu_l + n \rightarrow p + l^- ,$$

terms of upto sixth power in the laboratory energy of the incident neutrino would be present in the cross section- a clear deviation from the square dependence expected from a simple current-current interaction. In the absence of any information about the coupling of such a resonance to the leptons and the nucleon, the importance of the terms with higher powers of energy cannot be estimated. Albright and Good consider the appearance of a Breit-Weigner resonance structure in the cross section the dominant feature of this type of non-locality. Further, differences in total cross section for neutrino and antineutrino scattering off nucleons is expected. Both these features would of course be absent in the violations of lepton pair locality theorems due to a scalar boson mediated theory which we have discussed.

Deviations from the explicit lepton variable dependence of cross sections have also been considered by Albright[20], who points out that apparent deviations from the implications of a local current Lagrangian may signal the presence of a rapidly decaying heavy lepton coupling locally to the hadronic current in a fashion similar to the familiar leptons.

## Appendix A

## SCALAR BOSON MEDIATED WEAK INTERACTION THEORIES

Models of weak interactions in which the conventional weak interactions occur to the fourth order in an interaction mediated by scalar bosons were suggested by Kummer and Segre[6]. Such models were later rediscovered and extended by N. Christ[7]. The interaction term which consists of a product of two fermion fields and a spin zero field is an interaction of the first kind in the classification of Bogoliubov and Shirkov[18] and hence renormalizable. The couplings are written in a form which is invariant under chiral transformation on each of the familiar fermion fields like $e$, $\mu$, $\nu$, etc. In the limit in which the vertices are reduced to points, therefore, only a V-A form survives. These models were most successful in obtaining the conventional effective lagrangian to order $g^4$ for the purely leptonic reactions. We consider here the $\mu$ decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$.

The relevant part of the Lagrangian is (for consistency, we have taken the leptonic part also from the Gupta-Patil version).

$$\mathcal{L}_\ell = g \left[ W^- \sum_{l=e,\mu} L_l^-(1-\gamma_5)\,\nu_l + W^0 \sum_{l=e,\mu} L_l^-(1-\gamma_5) l^- \right] \quad (A1)$$

The matrix element for $\mu^-(p_1) \to \nu(p_2) + e^-(q_2) + \bar{\nu}_e(q_1)$ is

$$T_{fi} = \frac{g^4}{i(2\pi)^4} \int d^4q\ \bar{u}(p_2)\ (1+\gamma_5)\ \frac{\gamma\cdot(-q+p_1)+M}{(-q+p_1)^2-M^2}\ (1-\gamma_5)\ u(p_1)$$

$$\times\ \bar{u}(q_2)(1+\gamma_5)\ \frac{\gamma\cdot(-q+q_2)+M}{(-q+q_2)^2\ -M^2}\ (1-\gamma_5)\ u(q_1)\ \frac{1}{(q^2-\mu^2)}\cdot\frac{1}{(q-L)^2-M^2}$$

$$\ldots \text{(A2)}$$

$$= \frac{4g^4}{i(2\pi)^4}\ \bar{u}(p_2)\ \gamma^{\lambda}\ (1-\gamma_5)u(p_1)\ \bar{u}(q_2)\ \gamma^{\sigma}\ (1-\gamma_5)u(q_1)\ \int(q-p_1)_{\lambda}$$

$$\frac{(q-q_2)_{\sigma}\quad d^4q}{[(q-p_1)^2-M^2]\ [(q-q_2)^2-M^2]\ [(q-L)^2-M^2](q^2-\ \mu^2)}$$

$$\ldots \text{(A3)}$$

In the integral only the part containing $q_{\lambda}\ \ q_{\sigma}$ gives contribution to the required order. The integral of interest is, therefore,

$$I_{\mu\lambda} = \int \frac{q_{\lambda}\ q_{\sigma}\ \ d^4q}{[(q-p_1)^2-M^2][(q-q_2)^2-M^2]\ [(q-L)^2-M^2](q^2-\ \mu^2)}$$

$$= 6\int_0^1 x^2y\ dx\ dy\ dz \int \frac{q_{\lambda}\ q_{\sigma}\ d^4q}{[q^2 + 2q.p + s + i\varepsilon]^4}$$

where $\quad q^2+2q.p+s+i\varepsilon = [(q-p_1)^2-M^2+i\varepsilon]\ xyz + [(q-q_2)^2-M^2+i\varepsilon]$

$$xy(1-z) + [(q-L)^2-M^2+i\varepsilon]\ x(1-y) + (q^2-\ \mu^2+i\varepsilon\ )(1-x)$$

$$= q^2+2q.\ [-p_1xyz\ -q_2xy(1-z)\ -Lx(1-y)]$$

$$-M^2x\quad -\ \mu^2(1-x)\ +m_{\mu}{}^2\ xyz\ +m_e{}^2\ xy(1-z)\ +\ tx(1-y)\ +\ i\varepsilon\ .$$

Now $$\int \frac{d^4q \quad q_\lambda \, q_\sigma}{(q^2+2q.p+s+i\epsilon)^4} = \frac{i\pi^2}{12}\left[\frac{g_{\lambda\sigma}}{(p^2-s)} - \frac{2p_\lambda \quad p_\sigma}{(p^2-s)^2}\right]$$

only coefficient of $g_{\lambda\sigma}$ has terms to order $1/M^2$. The coefficient of $g_{\lambda\sigma}$ in $I_{\lambda\sigma}$ is

$$6 \cdot \frac{i\pi^2}{12}\int_0^1 x^2y \; dxdydz \quad \frac{1}{\left[M^2x + \left[p_1 \; xyz + q_2 \; xy(1-z) + Lx(1-y)\right]^2 + \mu^2(1-x) - m_\mu{}^2 \; xyz - m_e{}^2 \; xy(1-z) - tx(1-y)\right]}$$

$$= \frac{i\pi^2}{2}\left[\frac{1}{M^2}\int_0^1 xy \; dxdydz + 0\left(\frac{1}{M^4}\right)\right]$$

$$= \frac{i\pi^2}{8M^2} + 0\left(\frac{1}{M^4}\right)$$

Substituting terms to order $\frac{1}{M^2}$ in (A3)

$$T_{fi} = \frac{g^4}{32\pi^2M^2}\,\bar{u}(p_2)\,\gamma^\lambda(1-\gamma_5)\,u(p_1)\,\bar{u}(q_2)\gamma_\lambda(1-\gamma_5)\,u(q_1)$$

$$= \frac{G}{\sqrt{2}}\,\bar{u}(p_2)\;\gamma^\lambda(1-\gamma_5)\,u(p_1)\,\bar{u}(q_2)\;\gamma_\lambda(1-\gamma_5)\,u(q_1)$$

where $\frac{G}{\sqrt{2}} = \frac{g^4}{32\pi^2M^2}$

Therefore, to the lowest order in $\frac{1}{M^2}$ (A1) gives same matrix elements as the four fermion effective Lagrangian.

Apart from the usual leptonic interactions, corrections are expected to the anomalous magnetic moments of the charged leptons. The contribution to the anomalous magnetic moment

of the muon is $(g-2)_{\text{muon}} \approx \dfrac{g^2 m_\mu^{\ 2}}{24\pi^2 M^2}$ .

If $M > 1\text{GeV}$ this is small enough to be unobservable at present.

Semileptonic weak interactions can also be described along similar lines. Here instead of completely specifying the hadronic weak currents, only the Bjorken limit may be retained for the time ordered product of the hadronic scalar and pseudoscalar currents, whose commutators are assumed to be the same as in the quark model. A V-A form is thereby obtained for semileptonic reactions.

However, in describing the weak interactions of hadrons the earlier versions ran into difficulties. With only one set of $W^{\pm}$ and $W^{o}$ , parity and strangeness violations to order $g^2$ can occur through $\alpha \rightarrow \beta + W \rightarrow \alpha$. N. Christ introduces additional fermions and bosons so that the couplings conserve strangeness. But parity violations to order $g^2$ cannot be as simply dealt with. Christ points out that the nature of the parity violations also depends on the strong interaction Hamiltonian and one may have situations where the parity violations may be reduced to within the experimentally acceptable levels.

Patil and Vaishya[8] pointed out that these difficulties can be overcome by coupling the intermediate bosons to the

leptons and the hadrons with different coupling strengths. However, the scalar and pseudoscalar currents were assumed to transform like a SU(3) triplet and this implies one needs exotic hadrons.

The neatest version of a scalar boson mediated weak interaction theory, as mentioned in Chapter I, is due to Gupta and Patil[9]. Here the hadronic scalar and pseudoscalar currents are assumed to have octet transformation properties. Further, the neutral intermediate boson is identified with a combination of the neutral pseudoscalar mesons $\pi^0$, $\eta^0$ and $X^0$. Gupta and Patil, therefore, have a model with 'less than the minimum' number of additional particles. The Lagrangian of this model has been given in Chapter I. We give here some further details concerning the model.

A universality criterion fixes the various hadronic and leptonic coupling constants in the model in terms of a single undetermined coupling constant, which can be fixed in terms of the fermi coupling constant by considering the $\mu$ decay. The axial vector and vector charge densities generated by commutation among the scalar and pseudoscalar currents are identified with charge densities of the V-A currents occurring in the effective current-current Lagrangian. Thus the charges corresponding to the leptonic currents generate the usual leptonic weak charge if $g' = g$ and the commutators

the hadronic scalar and pseudoscalar currents give rise to can be identified with the hadronic charges of the V and A currents occurring in the effective weak Lagrangian if
$g_+ g_\pi = g^2$.

The requirement that the neutral hadronic currents be suppressed to within experimental limits gives limits on the mass M of the scalar boson. For the decay $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ the ratio calculated is

$$r = \frac{R(K^+ \rightarrow \pi^+ \nu\bar{\nu})}{R(K^+ \rightarrow \pi^0 e^+ \nu_e)} = \frac{4 g_+^2 \cos^2 \theta_c}{g_\pi^2}$$

With the experimental bound of $r < 2.4 \times 10^{-5}$ one obtains $M \leq 20$ GeV.

Further parity violation in ep and $\mu$p scattering is expected in this model. In the leading Bjorken limit the contribution is zero since the neutral intermediate bosons are their own antiparticles. The contribution is estimated with a nucleon intermediate state. For momentum transfers of $Q^2 = -m_N^2$ this is of the order of $\frac{1}{200M^2} \ln\left(\frac{M^2}{Q^2}\right)$ which is small enough to be unobservable at present with the expected values of M.

Gupta and Patil suggest that improvement of the experimental bounds may provide a test for their model. Further

direct test will involve production of massive leptons and the scalar bosons for which the suggested reactions are

$$p\bar{p} \rightarrow l^{\pm} L_l^{\pm}$$

and

$$p\bar{p} \rightarrow \pi^{\mp} W^{\pm}$$

The heavy lepton will probably decay through

$$L_l^{\pm} \rightarrow \pi^0 l^{\pm}$$

which may be distinguished from $p\bar{p} \rightarrow \pi^0 l^{\pm} l^{\mp}$ by looking at mass distribution or from lepton polarization. The boson may similarly decay through

$$W^+ \rightarrow p\,\bar{n}$$
$$\rightarrow \pi^0\, l\, \nu_l$$

The decay rates for the $W$ are much smaller than for the heavy lepton and tests for its detection may, therefore, be easier.

# Appendix B

## KINEMATICS AND PHASE SPACE

In this appendix we give kinematic and phase space details for expressions in Chapters II and IV.

### B.1 KINEMATIC DETAILS FOR CHAPTER II

We derive here expressions for the components of the four vectors involved in obtaining the lepton pair locality theorems of Pais and Treiman. As mentioned in the text, we work in the laboratory frame. Labeling of the various vectors and choice of kinematic variables is as in the text. Four momentum conservation given by $q_1 + p = q_2 + P$ where $P$ $P = p_1 + p_2 + \cdots + p_n$ , is used at various stages.

a) Components of L

$$L_o = q_{2o} - q_{1o} = (p-P)_o = \frac{1}{2m_N}(2m_N^2 - 2P.p)$$

$$= \frac{1}{2m_N}\left[ m_N^2 + (p-P)^2 - P^2 \right] = \frac{1}{2m_N}(m_N^2 + t-s) = \frac{Y}{m_N}$$

$$L_3 = (L_o^2 - t)^{1/2} = \frac{1}{m_N}(Y^2 - m_N^2 t)^{1/2}$$

$$L_1 = L_2 = 0$$

b) Components of N

$$N_o = q_{2o} + q_{1o} = 2q_{1o} + L_o = 2E + L_o$$

$$N.L = (q_2 + q_1)\cdot(q_2 - q_1) = m_1^2$$

$$-N_3L_3 + N_oL_o = m_1^2$$

$$N_3 = \frac{1}{L_3} \left( -m_l^2 + N_o L_o \right)$$

$$= \frac{m_N}{(Y^2 - m_N^2 t)^{1/2}} \left[ -m_l^2 + \left(2E + \frac{Y}{m_N}\right) \frac{Y}{m_N} \right]$$

$$= \frac{1}{m_N (Y^2 - m_N^2 t)^{1/2}} \left( Y^2 + 2E m_N Y - m_l^2 m_N^2 \right)$$

$N_1 = n \operatorname{Cos} \phi \qquad N_2 = n \operatorname{Sin} \phi$

$n$ obtained from $\quad N^2 = -t + 2m_l^2$

$$\text{i.e.,} N_o^2 - N_3^2 - n^2 = -t + 2m_l^2$$

$$n^2 = t - 2m_l^2 + N_o^2 - \frac{1}{L_3^2} (-m_l^2 + N_o L_o)^2$$

$$n^2 = t - 2m_l^2 - \frac{m_N^2}{(Y^2 - t m_N^2)} \left[ t N_o^2 + m_l^4 - 2m_l^2 N_o L_o \right].$$

## B.2 PHASE SPACE EXPRESSIONS FOR CHAPTER II

The reaction is $\bar{\nu}_l(q_1)(\text{or } \nu_l) + p(p) \rightarrow \alpha(P) + l^+(q_2)(\text{or } l^-)$

$$E' = (q_{2o})_{lab}.$$

$$= \frac{1}{|v_1 - v_2|} \int \frac{1}{2m_N} \frac{1}{2E} \frac{m_l}{E'} \frac{d^3 q_2}{(2\pi)^3} \frac{d^3 p_1}{(2\pi)^3 2p_{1o}} \cdots \frac{d^3 p_n}{(2\pi)^3 2p_{no}} T_{\mu\nu} \tau^{\mu\nu}$$

$$\times (2\pi)^4 \delta(p + q_1 - P - q_2)$$

$|v_1 - v_2| = 1$

$$= \frac{m_l}{4 m_N E (2\pi)^3} \int \frac{d^3 q_2}{E'} \int ds\, \delta(P^2 - s)\, d^4 P\, \delta(P - p_1 \ldots - p_n)$$

$$\times \frac{d^3 p_1}{(2\pi)^3 2p_{1o}} \cdots \int \frac{d^3 p_n}{(2\pi)^3 2p_{no}} T_{\mu\nu} \tau^{\mu\nu} \delta(p + q_1 - P - q_2)(2\pi)^4 .$$

$$= \frac{m_1}{4m_N E\ (2\pi)^3} \int \frac{d^3q_2}{E'} \int ds\ \delta(P^2-s)\ d^4P\ \delta(p+q_1-P-q_2) J_{\mu\nu}\ \tau^{\mu\nu}$$

where $$J_{\mu\nu} = \int \frac{d^3p_1}{(2\pi)^3\ 2p_{1o}} \cdots \int \frac{d^3p_n}{(2\pi)^3\ 2p_{no}} (2\pi)^4\ \delta(P-p_1\ldots p_n) \times T_{\mu\nu}$$

integrating over P

$$\sigma = \frac{m_1}{4m_N E(2\pi)^3} \int \frac{d^3q_2}{E'} \int ds\ \delta([q_1+p-q_2]^2-s)\ J_{\mu\nu}\ \tau^{\mu\nu}$$

$$\frac{d\sigma}{ds dt d\phi} = \frac{m_1}{4m_N E(2\pi)^3} \int \frac{d^3q_2}{E'}\ \delta[t+m_N^2 + 2m_N(E-E')-s]$$

$$\delta[t-2E'(E'-E) + 2\cos\theta_e\ |\vec{q}_2| L_3 + m_1^2]\ \delta(\phi_e-\phi)\ J_{\mu\nu}\ \tau^{\mu\nu}$$

since $t = (q_2-q_1)^2 = -2q_1 . q_2 + m_1^2 = 2q_2.(q_2-q_1)-m_1^2 = 2q_{2o}L_o - 2(q_2)_3\ L_3 - m_1^2$

$$= 2E'(E'-E) - 2\cos\theta_e\ |\vec{q}_2|\ L_3 - m_1^2$$

also since $L_1 = L_2 = 0$ $\quad (q_1)_1 = (q_2)_1 = \frac{1}{2} n \cos\phi = \frac{1}{2} N_1$

$\quad (q_1)_2 = (q_2)_2 = \frac{1}{2} n \sin\phi = \frac{1}{2} N_2$

$\phi = \phi_e$

$$\frac{d\sigma}{ds dt d\phi} = \frac{m_1}{4m_N E(2\pi)^3} \int |\vec{q}_2|\ dE'\ \sin\theta_e\ d\theta_e\ d\phi_e\ \delta[s-m_N^2 t-2m_N(E-E')]$$

$$\delta[t-2E'(E'-E) + 2\cos\theta_e |\vec{q}_2| L_3 + m_1^2]\ \delta(\phi-\phi_e)\ J_{\mu\nu}\ \tau^{\mu\nu}$$

integrations over $E'$, $\theta_e$, $\phi_e$ are now performed.

$$\frac{d}{ds\,dt\,d\emptyset} = \frac{m_l}{128\pi^3 m_N L_3 E} J_{\mu\nu}\,\tau^{\mu\nu}$$

## B.3 PHASE SPACE EXPRESSIONS FOR CHAPTER IV

$$\bar{\nu}_l(q_1) + p(p) \rightarrow \alpha(P) + l^+(q_2) + \gamma(k)$$

$$E' = (q_{2o})_{lab}$$

$$\sigma = \frac{1}{|v_1-v_2|} \int \frac{1}{2m_N} \frac{1}{2E} \frac{d^3q_2}{(2\pi)^3} \frac{m_l}{E} \frac{d^3k}{(2\pi)^3 2k_o} \frac{d^3p_1}{(2\pi)^3 2p_{1o}} \cdots \frac{d^3p_n}{(2\pi)^3 2}$$

$$x\ (2\pi)^4\ \delta(q_1 + p-k-p_1 \ldots -p_n) \sum_{spins} |T|^2$$

$$|v_1-v_2| = 1$$

$$= \frac{m_l}{(2\pi)^2\, 8m_N E} \int \frac{d^3q_2}{E'} \frac{d^3k}{k_o} \int dw\ \delta(P^2-w)\ d^4P\ \delta(q_1+p-k-q_2-P) I$$

where $I = \int \frac{d^3p_1}{(2\pi)^3 2p_{1o}} \cdots \int \frac{d^3p_n}{(2\pi)^3 2p_{no}} \Sigma|T|^2\ \delta(P-p_1 \ldots -p_n)$

performing integration over $P$

$$= \frac{m_l}{(2\pi)^2\ 8m_N E} \int dw \frac{d^3q_2}{E'} \frac{d^3k}{k_o} \delta\left[(q_1-q_2-k-p)^2-w\right] I$$

$$(p-k-L)^2-w = m_N^2 + t - 2m_N L_o - w - 2k_o\ (m_N - L_o + L_3 \cos\theta_k)$$

integrating over $\theta_k$

$$= \frac{m_l}{(2\pi)^2\ 8m_N E} \int \frac{d^3q_2}{E'} dk_o\, d\emptyset_k \frac{1}{-2L_3} I dw$$

$$\text{since} \quad s = (k+P)^2 = (p-L)^2 = m_N{}^2 + t + 2m_N(E-E')$$

$$t = (q_2-q_1)^2 = -2q_1 \cdot q_2 \qquad = 2q_2 \cdot (q_2-q_1)$$

$$= 2q_{2o}\, L_o - 2(q_2)_3 L_3 = 2E'(E'-E) - 2 \operatorname{Cos} \theta_e \, |\vec{q}_2| L_3$$

also since $L_1 = L_2 = 0$ $\quad \frac{1}{2} N_1 = (q_1)_1 = (q_2)_1 = \frac{1}{2} n \operatorname{Cos} \phi$

$$\frac{1}{2} N_2 = (q_1)_2 = (q_2)_2 = \frac{1}{2} n \operatorname{Sin} \phi$$

$$\phi_e = \phi$$

$$\frac{d\sigma}{dwdsdtdk_o d\phi} = \frac{m_1}{-2L_3(2\pi)^2 \; 8m_N E} \int |\vec{q}_2| \, dE' \operatorname{Sin} \theta_e \, d\theta_e \, d\phi_e \, d\phi_k \, I$$

$$x\ \delta \left[ s - m_N{}^2 - t - 2m_N(E-E') \right] \delta \left[ t - 2E'(E'-E) + 2 \operatorname{Cos} \theta_e |\vec{q}_2| L_3 \right] \delta(\phi_e - \phi)$$

Integration over $\phi_k$ gives a $2\pi$ factor. Integrate over $E'$, $\theta_e$ and $\phi_e$ using the first, second and third delta functions.

$$\frac{d\sigma}{dsdtdwdk_o d\phi} = \left( \frac{m_1}{-128\pi m_N{}^2 L_3{}^2 E} \right) I$$

## $\phi$ Integration

In performing integration over $\phi$ to obtain the energy dependence theorem, one has integrands of the form $\frac{\operatorname{Cos}^m \phi \operatorname{Sin}^n \phi}{q + r \operatorname{Cos} \phi}$ where $m$ and $n$ are integers such that $m + n \leq 3$.

## Appendix C

## APPENDIX FOR CHAPTER III

In this section we perform a more careful evaluation of the integrals of Chapter III. In particular we show that expansion in inverse powers of $M^2$ is permitted when $2m_N E < M^2$. We consider the integral occurring as the coefficient of $g_{\mu\lambda}$ in $I_{\mu\lambda}$. The coefficient of the second term in $I_{\mu\lambda}$ may be similarly evaluated.

$$I^1_{\mu\lambda} = g_{\mu\lambda} \int_0^1 x^2 dx \int_0^1 y dy \int_0^1 dz \Big[ M^2 xy - N.p_1 \; xyz(1-x) + 2L.q_2 \; xyz \\ (1-x+xy-xyz) - 2L.p_1 (1-x) + t\big( (1-x+xy-xyz)^2 - xy + xyz \\ +(1-x) \big) + \mu^2 x(1-z) \Big]^{-1}.$$

We assume $M^2 > N.p_1 = 2Em_N + m_N L_o \gg t, \mu^2$.

$$I^1_{\mu\lambda} = g_{\mu\lambda} \int_0^1 x^2 dx \int_0^1 y dy \int_0^1 dz \frac{1}{\left[ M^2 xy - N.p_1 \; xyz(1-x) \right]}$$

Coefficient of $g_{\mu\lambda}$ is

$$\int_0^1 x dx \int_0^1 dy \int_0^1 dz \quad \frac{1}{M^2 - N.p_1 z(1-x)}$$

$$= \int_0^1 x dx \int_0 dz \frac{1}{M^2 - N.p_1 z(1-x)}$$

$$= -\frac{1}{N.p_1} \int_0^1 \frac{x dx}{1-x} \log \left[ 1 - \frac{N.p_1 (1-x)}{M^2} \right]$$

Changing variable $x \to 1-x$

$$= -\frac{1}{N.p_1}\int_0^1 \frac{(1-x)dx}{x} \log\left(1 - \frac{N.p_1 x}{M^2}\right)$$

$$= \frac{1}{N.p_1}\left[\int_0^1 dx \log\left(1 - \frac{N.p_1 x}{M^2}\right) - \int_0^1 \frac{dx}{x}\log\left(1 - \frac{N.p_1 x}{M^2}\right)\right]$$

$$= \frac{1}{N.p_1}\left[\frac{1-N.p_1/M^2}{-N.p_1/M^2}\log\left(1-\frac{N.p_1}{M^2}\right) - 1 - \int_0^1 \frac{dx}{x}\log\left(1-\frac{N.p_1 x}{M^2}\right)\right]$$

$$= \frac{1}{N.p_1}\left[\left(1-\frac{M^2}{N.p_1}\right)\left[-\frac{N.p_1}{M^2} - \frac{1}{2}\left(\frac{N.p_1}{M^2}\right)^2 - \frac{1}{3}\left(\frac{N.p_1}{M^2}\right)^3 \cdots\right] - 1\right.$$

$$\left. - \int_0^1 \frac{dx}{x}\log\left(1-\frac{N.p_1 x}{M^2}\right)\right]$$

$$= \frac{1}{N.p_1}\left[-\frac{1}{2}\frac{N.p_1}{M^2} - \frac{1}{6}\left(\frac{N.p_1}{M^2}\right)^2 + 0\left(\frac{1}{M^6}\right) + \int_0^1 \frac{dx}{x}\left[\frac{N.p_1 x}{M^2} + \frac{(N.p_1 x)^2}{2M^4}\right]\right].$$

Logarithmic expansions in both terms are permitted since $1 > \frac{N.p_1}{M^2} \geq \frac{N.p_1 x}{M^2}$ for $0 < x < 1$.

$$= \frac{1}{N.p_1}\left[\frac{1}{2}\frac{N.p_1}{M^2} + \frac{(N.p_1)^2}{12M^4} + 0\left(\frac{\overline{N.p_1}^3}{M^6}\right)\right]$$

$$I^1_{\mu\lambda} = g_{\mu\lambda}\left[\frac{1}{2M^2} + \frac{N.p_1}{12M^4} + 0\left(\frac{1}{M^6}\right)\right]$$

$I^1_{\mu\lambda}$ is retained to $\frac{1}{M^4}$ terms in our calculation.

$$I^1_{\mu\lambda} = g_{\mu\lambda}\left[\frac{1}{2M^2} + \frac{N.p_1}{12M^4}\right].$$

## Appendix D

## VIOLATION OF THE ANGLE THEOREM IN $\bar{\nu}+p\to n+\mu^{+}+\eta$

In this appendix we obtain expressions for violation of the angle theorem for a simple inelastic process $\bar{\nu}(q_1) + p(p_1) \to n(p_2) + \mu^{+}(q_2) + \eta(k)$. Kinematics is along the lines described in Chapter II. The hadronic vector $\vec{Q}$ of Chapter II is identified with $\vec{k}$. An additional kinematic variable, in addition to those of the quasielastic reaction, is chosen to be $\theta$, the angle between $\vec{k}$ and the $z$ axis.

As mentioned in the text, violation of the theorem comes only from Figure 2(c). The amplitude for this is

$$T_c = \frac{2g^4 \operatorname{Cos}\theta_c}{(2\pi)^4 i}\, g_{NN\eta}\; \bar{v}(q_1)\gamma_\lambda(1-\gamma_5)v(q_2)$$

$$\times \int \frac{d^4q\; q^\lambda}{(q^2-\mu^2)[(q-q_2)^2-M^2][(q-L)^2-M^2]}$$

$$\times\; \bar{u}(p_2)\;\gamma_5\; \frac{\gamma.(q+p_1-k-L)-m_N}{(q+p_1-k-L)^2-m_N{}^2}\,(1-\gamma_5)\; \frac{\gamma.(p_1-k)-m_N}{(p_1-k)^2-m_N{}^2}\,\gamma_5\; u(p_1)$$

$$= \frac{2g^4 \operatorname{Cos}\theta_c}{(2\pi)^4 i}\, g_{NN\eta}\quad l^\lambda\; \bar{u}(p_2)\;\gamma^\lambda I^{(1)}_{\mu\lambda}\;\;(1-\gamma_5)\; S_F(p_1-k)u(p_1)$$

where

$$I^{(1)}_{\mu\lambda} = \int \frac{d^4q\; q_\mu\; q_\lambda}{(q^2-\mu^2)[q-q_2)^2-M^2][(q-L)^2-M^2][(q-p_1-k-L)^2-m_N{}^2]}$$

$I^{(1)}_{\mu\lambda}$ is obtained by replacing $p_1$ by $p_1-k$ in $I_{\mu\lambda}$ of the quasielastic reaction. Keeping terms to order $\frac{1}{M^4}$,

$$I^{(1)}_{\mu\lambda} = \frac{i\pi^2}{4M^2}\left[ g_{\mu\lambda}\left(1+\frac{N.(p_1-k)}{6M^2}\right) + \frac{N_\mu\,(p_1-k)_\lambda}{6M^2}\right]$$

$$T_c = \frac{G}{\sqrt{2}} g_{NN\eta}\; l^\lambda\; \bar{u}(p_1)\,\gamma^\sigma\,(1-\gamma_5)\,S_F\,(p_1-k)\gamma_5\,u(p_1)$$

$$\times\left[ g_{\lambda\sigma}\left[1+\frac{N.(p_1-k)}{6M^2}\right] + \frac{N_\sigma\,(p_1-k)_\lambda}{6M^2}\right]$$

Squaring the amplitudes additional terms of Cos 3Ø are obtained from $\sum\limits_{\text{Spins}} |T_c|^2$ and $\sum\limits_{\text{Spins}} (T_c\,T_a^\dagger + T_a T_c^\dagger)$ where

$$T_a = \frac{G}{\sqrt{2}}\, g_{NN\eta}\; l^\lambda\; \bar{u}(p_2)\;\gamma_5\;S_F(p_2+k)\gamma_\lambda\,(1-\gamma_5)\,u(p_1).$$

As an estimate of the violation of the angle theorem we calculated R defined as the ratio of the coefficient of Cos 3Ø term in $\frac{d\sigma}{ds\,dt\,d\emptyset}$ as calculated in the scalar theory to that of the Cos 2Ø term with a simple current-current Lagrangian.

$$\frac{d\sigma}{ds\,dt\,d\emptyset} = \frac{m_l}{48L_3E}\,(t-2m_N L_o)\int \frac{\sin\theta\; d\theta}{(m_N-L_3\cos\theta)^2}\sum_{\text{Spins}} |T|^2$$

$$R(E,s,t) = \frac{\pi}{3}\,\frac{m_N s(-t)^{1/2}}{M^2}\cdot\frac{s-m_N^{\,2}}{s+m_N^{\,2}}\;\frac{\left[(4Em_N+x_1)^2-y_1^{\,2}\right]^{1/2} y_1^{-2}}{\frac{2x_1y_1}{x_1^{\,2}-y_1^{\,2}}+\log\left(\frac{x_1+y_1}{x_1-y_1}\right)}$$

where $x_1 = m_N^2 + s - t$ .  $y_1 = ( x_1^2 - 4tm_N^2 )^{1/2}$.

For a scalar boson of mass $M = 10$ GeV, this gives at $E = 30$ GeV for specific values of $s$ and $t$ mentioned, the following values.

| | $t = -2\ GeV^2$, $s = 40\ GeV^2$ | $t = -2\ GeV^2$, $s = 2\ GeV^2$ | $t = -40\ GeV^2$, $s = 2\ GeV^2$ |
|---|---|---|---|
| R x 100 | < 0.01 | 2.3 | < 0.1 |

This may be compared with the value 37.5 for violation of the energy theorem with the same mass and laboratory energy as listed in Table 1.

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